Basics of Lidar, part 1
Elastic Lidar

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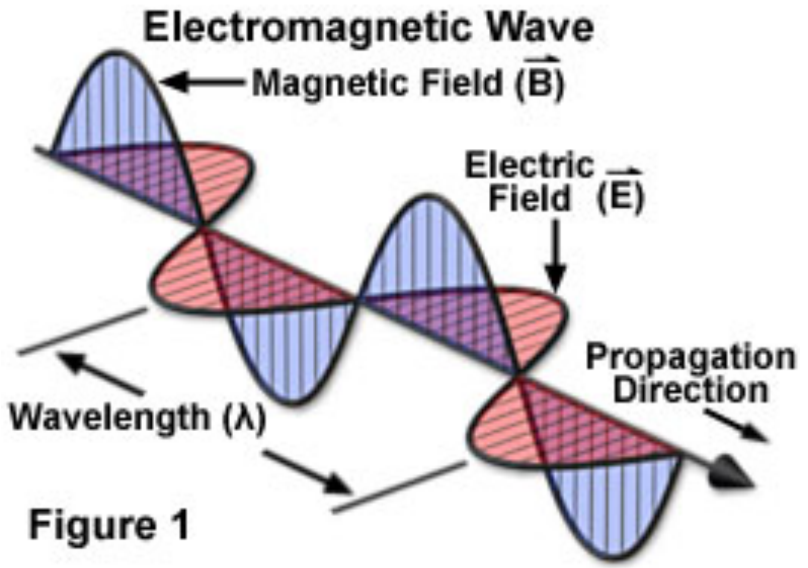
Outline

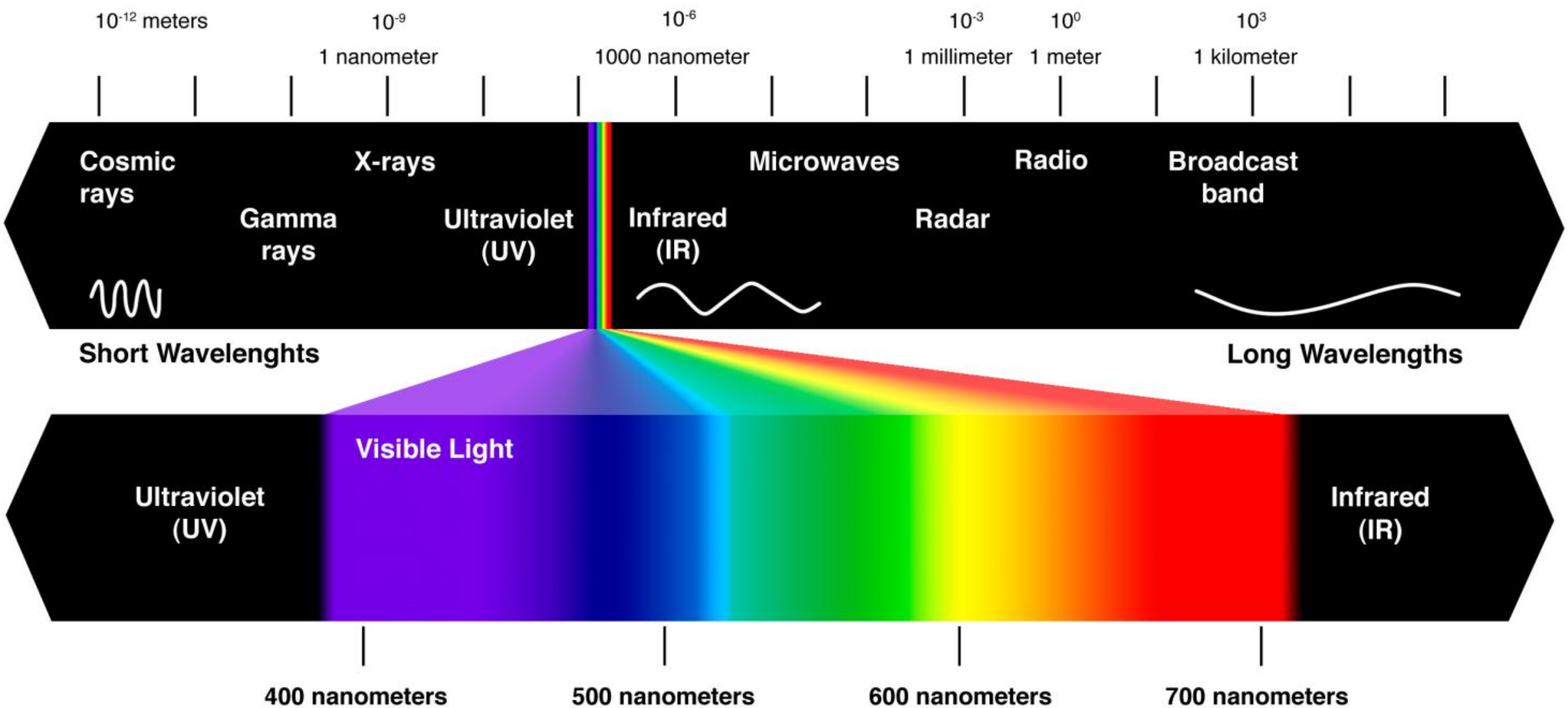
- **Radiative transfer in the atmosphere**
  - **Beer-Bouguer-Lambert law**
  - **Mie Scattering**
- Typical LIDAR setup
  - Detector
  - Overlap
- Lidar Equation
  - Molecular atmosphere
  - Klett-Fernald-Sasano
- The Lalinet (tentative) algorithm

Electromagnetic Radiation

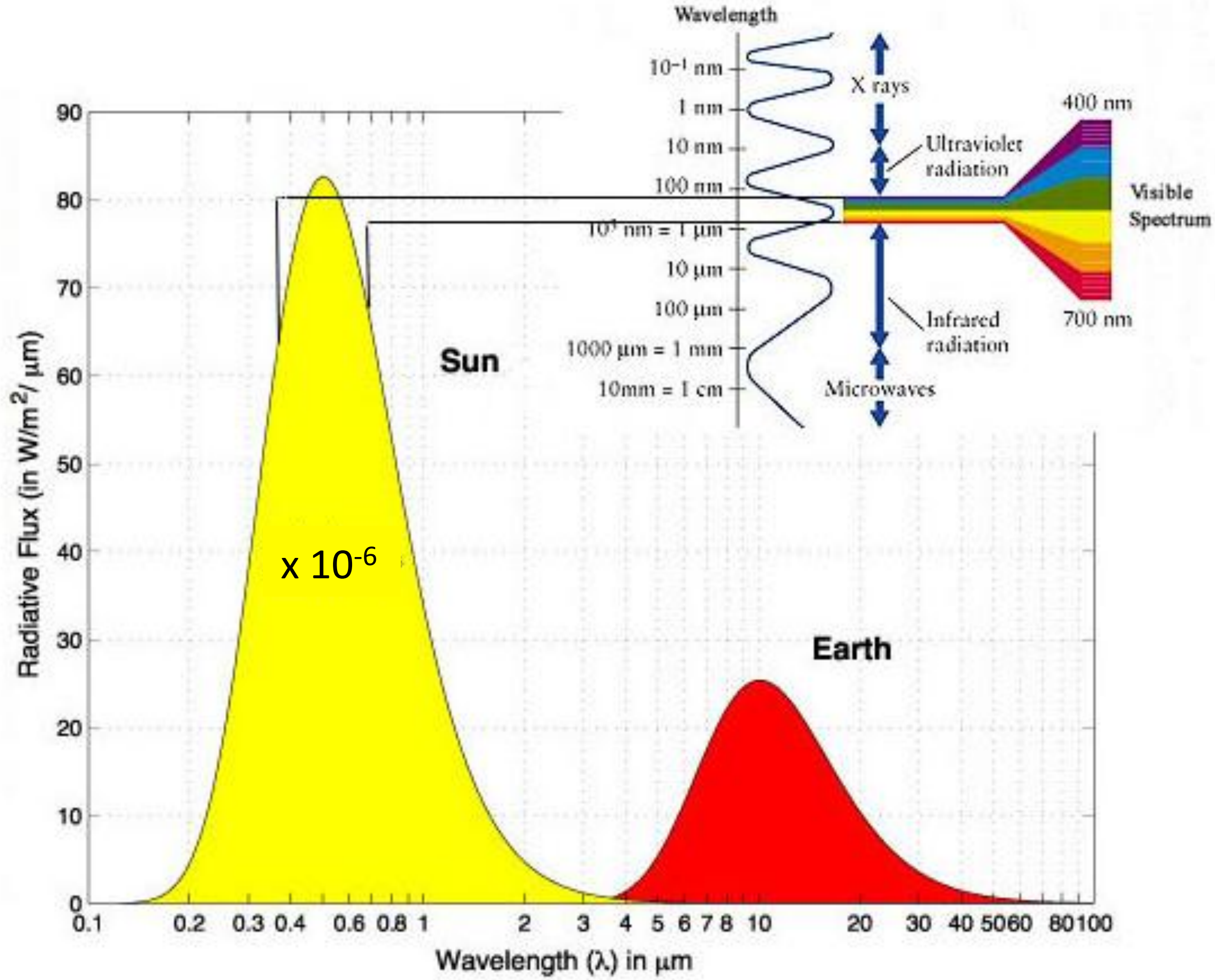
- It is a form of energy emitted and absorbed by charged particles, that propagates in space as a wave and as a particle at the same time;
- It is formed by a magnetic field and an electric field vibrating in phase, perpendicular to each other and to the propagating direction;
- The speed of propagation in vacum is a constant given by **E/B = c**



Eletromagnetic Spectrum

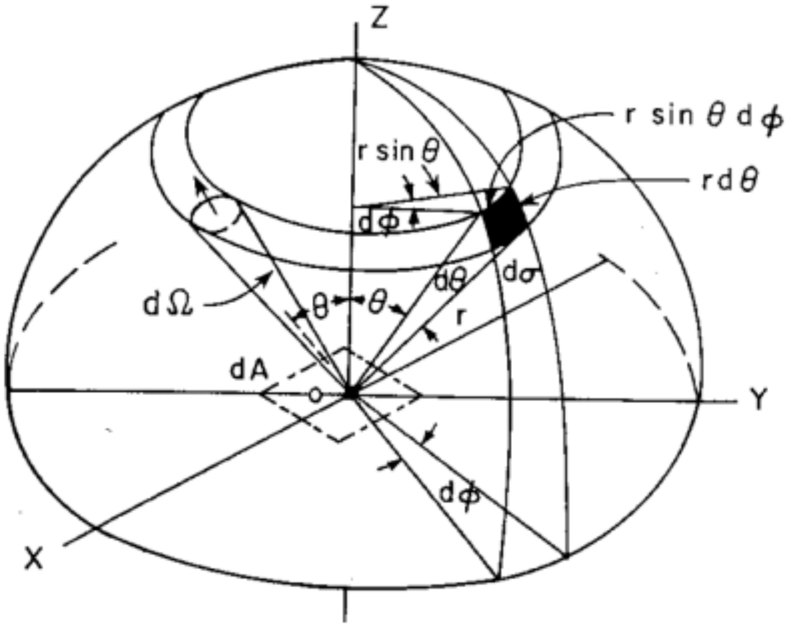


Solar and terrestrial spectrum



Radiance

- Radiant energy per unit time, per wavelength, per solid angle and per unit perpendicular area
  - Function of position, direction, wavelength and time

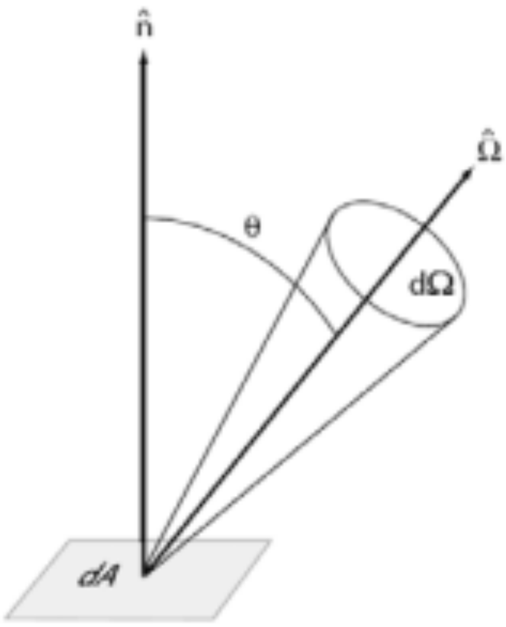


$$dI_\lambda(\vec{r},\hat{n},t) = \frac{dE_\lambda}{\cos\theta \cdot d\lambda \cdot da \cdot d\Omega \cdot dt}$$

Units: W m^{-2} sr^{-1} m^{-1}

Irradiance

- Radiant energy per unit time, per wavelength, per unit of perpendicular area
  - Integral of radiance over a certain solid angle



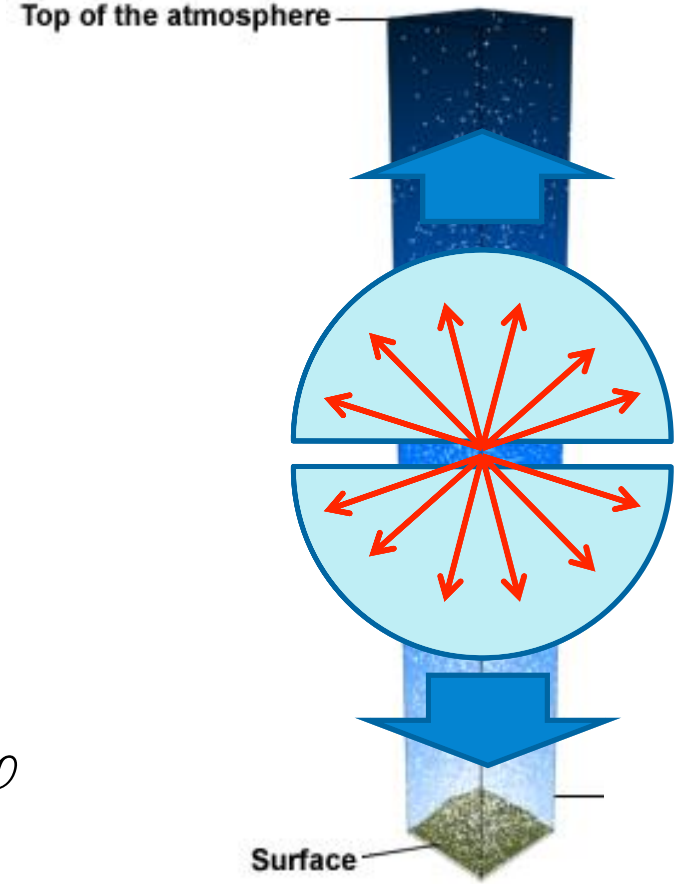
$$F_\lambda = \int_\Omega I_\lambda \cos(\theta)\, d\Omega$$

$$F_\lambda = \frac{dE_\lambda}{d\lambda \cdot da \cdot dt}$$

Units: W m^{-2} m^{-1}

Hemispheric Flux

- Integrating over the top half (upward flux) or lower half inferior (downward flux):



  - Upward:

$$F_\lambda^{\uparrow} = \int_0^{2\pi}\int_0^{\pi/2} I_\lambda(\theta,\varphi)\cos(\theta)\sin(\theta)\,d\theta\,d\varphi$$

  - Downward:

$$F_\lambda^{\downarrow} = -\int_0^{2\pi}\int_{\pi/2}^{\pi} I_\lambda(\theta,\varphi)\cos(\theta)\sin(\theta)\,d\theta\,d\varphi$$

Non-monocromatic Fluxes

- Downward shortwave radiation at the surface

$$SW_{sfc}^{\downarrow} = \int_{100\,nm}^{4\,\mu m} F_{\lambda}^{\downarrow}(sfc)\,d\lambda$$

- Outgoing longwave radiation:

$$LW_{top}^{\uparrow} = \int_{4\,\mu m}^{100\,\mu m} F_{\lambda}^{\uparrow}(top)\,d\lambda$$

- Downward PAR radiation:

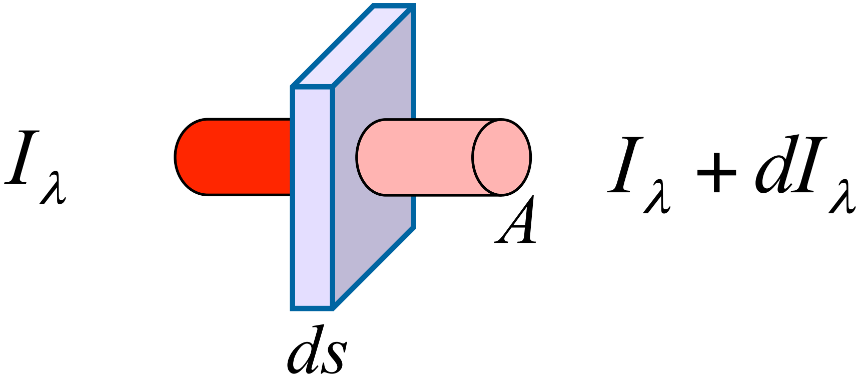
$$PAR = \int_{400\,nm}^{700\,nm} F_{\lambda}^{\downarrow}(sfc)\,d\lambda$$

Radiation in the Atmosphere

- Extinction and emission are the two main processes in the atmosphere
- Extinction:
  - It is a process that reduces the radiance. Can be due to **absorption** or to **scattering**
    - **Absorption: transforms EM energy in something else**
    - **Scattering: changes direction of propagation**
- Emission:
  - It is a process that increases the radiance.
    - All bodies with **T>0 K** emits radiation
    - There can be scattered radiation in the beam direction

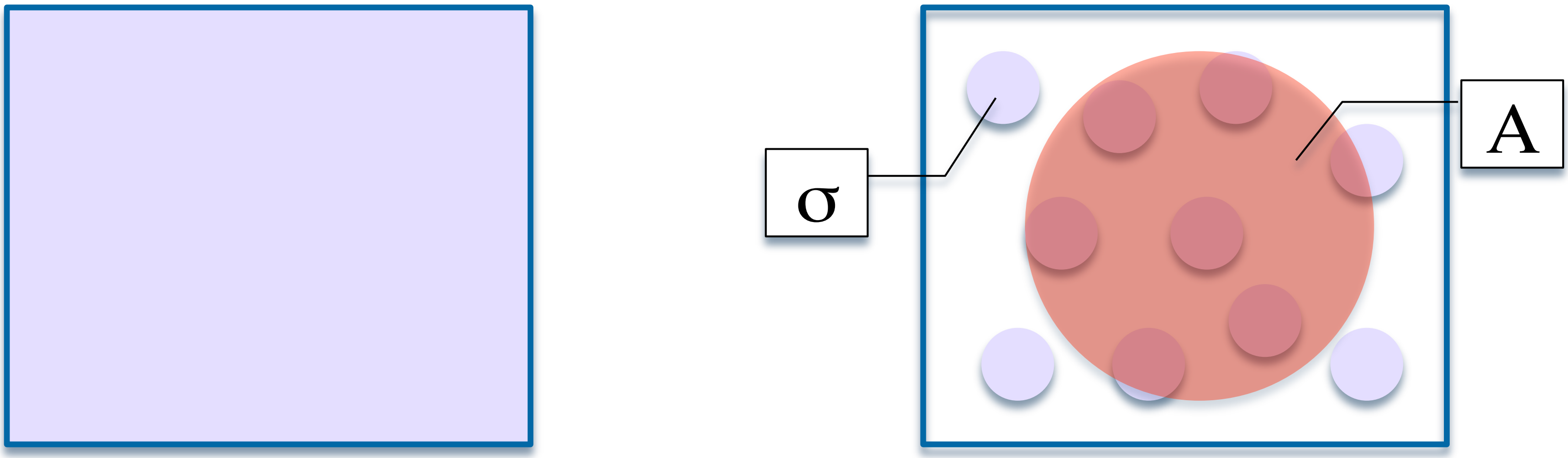
Light extinction

- The extinction process is proportional to the radiance and to the amount of matter



- If *ds* is small enough, there is no overlap between scatters (single layer limit)

Light extinction



$ds \rightarrow \infty$ $\qquad$ $ds \rightarrow 0$

- Therefore, the fraction of extinguished photons is

$$\frac{dI_\lambda}{I_\lambda} = -\frac{\sigma \cdot NAds}{A}$$

σ = cross section
N = # / volume
N A ds = #

Light extinction

- Therefore, in terms of extinction cross section [m^2], **σ**:

$$dI_\lambda = -\sigma N I_\lambda ds$$

- Or in terms of volume-extinction coefficient [1/m], **α**:

$$dI_\lambda = -\alpha I_\lambda ds$$

- Or in terms of mass extinction coefficient [m^2/kg]:

$$dI_\lambda = -(\alpha / \rho) I_\lambda d\chi \qquad d\chi = \rho \cdot ds$$

Mass thickness

- Or in terms of molar extinction [m^2/mol], **ε**:

$$dI_\lambda = -\varepsilon c I_\lambda ds$$

c = Molar concentration

Extinction along a path

- Solving this differential equation, we find:

$$I_\lambda(s) = I_\lambda(s_0)\exp\left[-\underbrace{\int_{s_0}^{s}\alpha(\lambda,s')ds'}\right]$$

- ... and if there is different types of particles:

$$I_\lambda(s) = I_\lambda(s_0)\exp\left[-\underbrace{\sum_i\int_{s_0}^{s}\alpha_i(\lambda,s')ds'}_{\tau(s_0,s)}\right]$$

Beer-Bouguer-Lambert Law

$$I = I_0 e^{-\tau}$$

- **Wikipedia** - *This law was discovered by Pierre Bouguer before 1729 and it is often (mis)attributed to Johann Heinrich Lambert, who cited Bouguer's "Essai d'Optique sur la Gradation de la Lumiere" (Claude Jombert, Paris, 1729), and even quoted from it, in his "Photometria" in 1760. Much later, August Beer extended the exponential absorption law in 1852 to include the concentration of solutions in the absorption coefficient.*

$$dI_\lambda / I_\lambda \propto ds$$

Bougher

$$dI_\lambda / I_\lambda \propto c$$

Beer

Emission of radiation

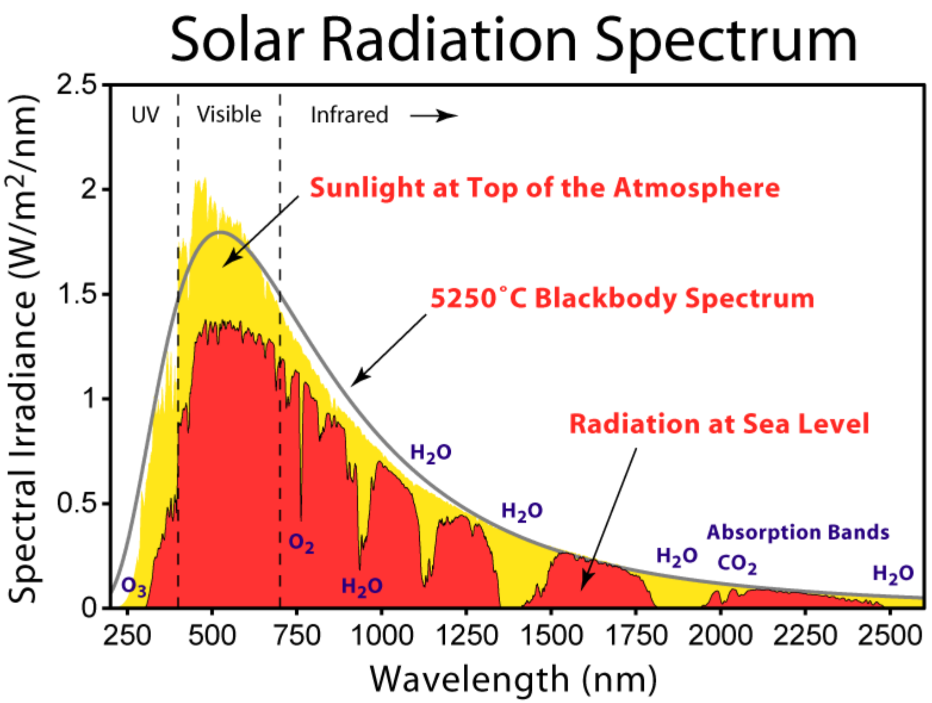
- Planck's function explains the radiance emitted by a black body (ideal):

$$B_\lambda = \frac{2\pi hc^2}{\lambda^5\left(e^{hc/kT\lambda} - 1\right)}$$

Units: W m⁻² sr⁻¹ m⁻¹

In the non ideal case, there is an emissivity:

$B_\lambda^{gray} = B_\lambda^{black} \cdot \varepsilon$



Stefan-Boltzmann law

- Integrating over all possible wavelengths, we find the total power emitted by the black body, which is the Stefan-Boltzmann radiation law:

$$P_{emitted} = \int_0^{\infty} B_\lambda d\lambda = \sigma T^4$$

- where σ=5.671 x 10^{-8} W m^{-2} K^{-4} is the Stefan-Boltzmann constant

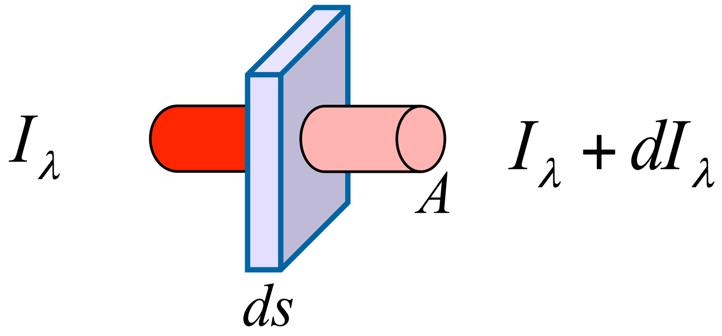
Kirchoff's Law

- If a body is in thermodynamical equilibrium, then:

Emissivity = Absorptivity

- Hence the **radiance** due to the emission of thermal radiation <u>has the same constant α</u>:

$$dI_\lambda = +\alpha B_\lambda ds$$



Schwarzchild's equation

- The radiative transfer equation is simply the beer-lambert law considering emission:

$$dI_\lambda = -\alpha I_\lambda ds + \alpha B_\lambda ds \Rightarrow \frac{dI_\lambda}{\alpha ds} = -I_\lambda + B_\lambda$$

- And our old solution … becomes more complicated

$$I_\lambda(s) = I_\lambda(s_0)e^{-\tau(s_0,s)} + \int_0^{\tau(s_0,s)} B_\lambda e^{-\tau'} d\tau'$$

- Schwarzschild's equation describes radiative transfer in a **nonscattering medium** (applies to remote sensing in the thermal IR band)

Scattering x Absorption

- We should remember that light can be extinguished by two processes, hence:

$$\alpha_{total} = \alpha_{abs} + \alpha_{scat}$$

- ... and that Kirchoff's law say $\alpha_{abs}=\alpha_{emis}$, hence the radiative transfer equation should be:

$$dI_{\lambda} = -\alpha_{scat} I_{\lambda} ds - \alpha_{abs} I_{\lambda} ds + \alpha_{abs} B_{\lambda} ds$$

- ... and that is why the previous solution is only valid for $\alpha_{scat}=0$

Points to remember #1

- We will see that we do not have to solve the radiative transfer equation in the case our a LIDAR system...
- What is important from this review is:

$$I(\lambda) = I_0(\lambda)\mathrm{e}^{-\tau_{total}}$$

$$\tau_{total}(\lambda) = \sum_{k=species} \int_{s_0}^{s} \alpha_{total}^{k}(\lambda, s')ds'$$

$$\alpha = N\sigma \Rightarrow \sigma_{total} = \sigma_{abs} + \sigma_{scat}$$

Extinction efficiency

- The cross-section has units of area (that "*shadows*" the light), but this "*area*" can be much larger or smaller then the real area, A_e, of the extinguisher.

$$\sigma_{ext} = \sigma_{abs} + \sigma_{scat} \qquad [L^2]$$

- We can define dimensionless ***scattering*** and ***absorbing efficiencies*** by making:

$$Q_{abs} = \frac{\sigma_{abs}}{A_e} \qquad Q_{scat} = \frac{\sigma_{scat}}{A_e}$$

Single scattering albedo

- The ratio of $\mathbf{Q_{scat}}$ and $\mathbf{Q_{ext}}$ is called the single-scattering albedo, ω, and represents the fraction of light extinction due to the scattering processes:

$$\omega = \frac{Q_{scat}}{Q_{ext}} = \frac{\sigma_{scat}}{\sigma_{ext}} = \frac{\sigma_{scat}}{\sigma_{scat} + \sigma_{abs}}$$

- Hence, **1-ω**, is the fraction that is absorbed.

Angstrom exponent

To obtain the extinction coefficient at the transmitted wavelength we have to introduce the Ångström exponent $\mathring{a}(R)$, which describes the wavelength dependence of the particle extinction coefficient,

$$\frac{\alpha_{\text{aer}}(\lambda_0)}{\alpha_{\text{aer}}(\lambda_{\text{Ra}})} = \left(\frac{\lambda_{\text{Ra}}}{\lambda_0}\right)^{\mathring{a}(R)}, \tag{4.18}$$

Table 4.1. Properties of aerosol types [1][a]

Aerosol type	N (cm^{-3})	r_{eff} (µm)	ssa (0.55 µm)	g (0.55 µm)	$\mathring{a}$ (0.35–0.55 µm)	$\mathring{a}$ (0.55–0.8 µm)
Cont. clean	2600	0.247	0.972	0.709	1.10	1.42
Cont. average	15,300	0.204	0.925	0.703	1.11	1.42
Cont. polluted	50,000	0.150	0.892	0.698	1.13	1.45
Urban	158,000	0.139	0.817	0.689	1.14	0.43
Desert	2300	1.488	0.888	0.729	0.20	0.17
Marit. clean	1520	0.445	0.997	0.772	0.12	0.08
Marit. polluted	9000	0.252	0.975	0.756	0.41	0.35
Marit. tropical	600	0.479	0.998	0.774	0.07	0.04
Arctic	6600	0.120	0.887	0.721	0.85	0.89
Antarctic	43	0.260	1.000	0.784	0.34	0.73
Stratosphere (12–35 km)	3	0.243	1.000	0.784	0.74	1.14

Weitkamp, chap 4

Light scattering mechanisms

Light scattering can be divided into three categories:

- **Elastic scattering** – when the wavelength of the scattered light does not change
- **Quasi-elastic scattering** – when the wavelength shifts slightly owing to Doppler effects and diffusion broadening
- **Inelastic scattering** – when the wavelength of the scattered radiation is different from the incident

Light scattering mechanisms

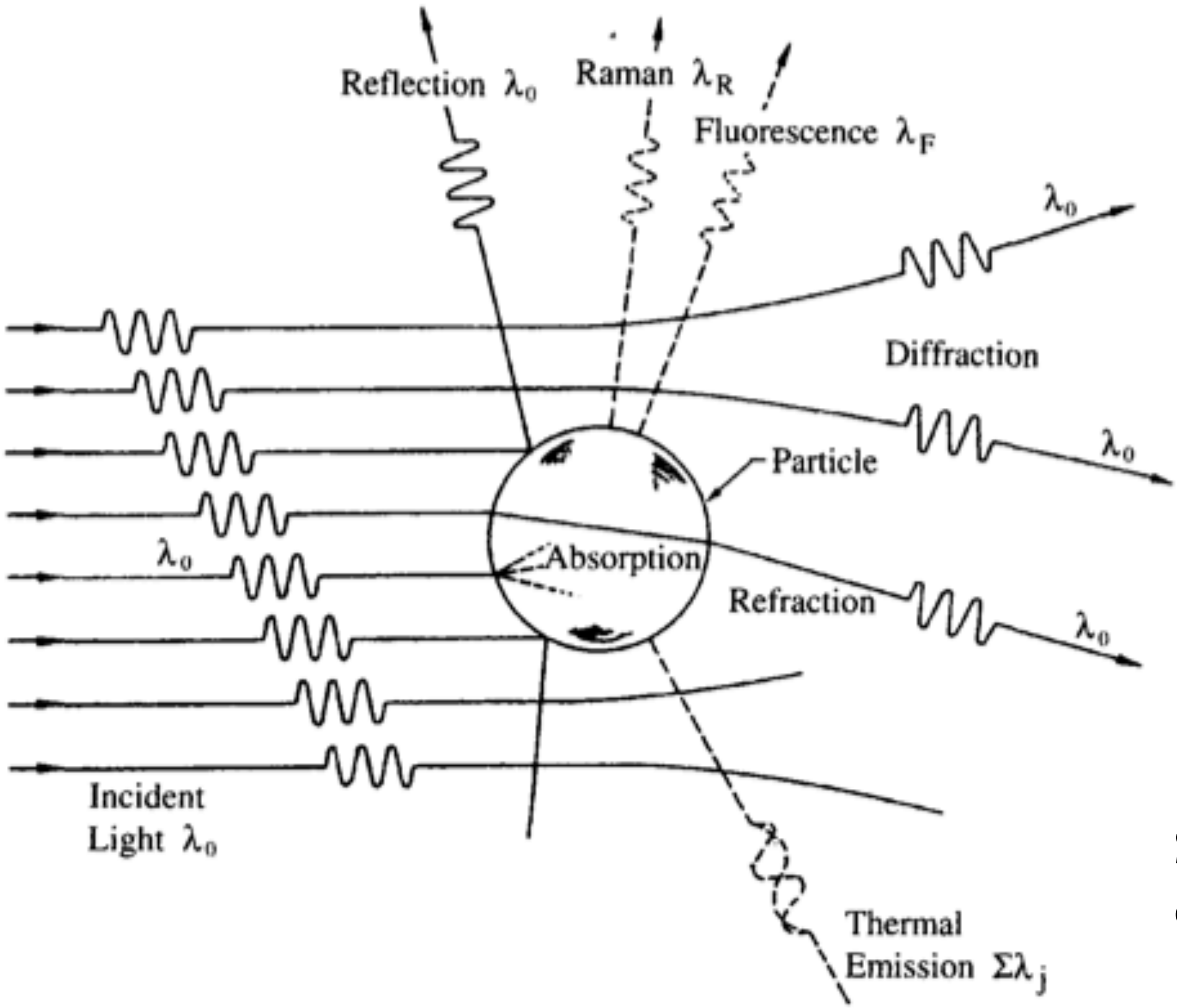


FIGURE 15.1 Mechanisms of interaction between incident radiation and a particle.

Seinfeld & Pandis, chapter 15

Light scattering

- Absorption and elastic scattering of light by a spherical object is a classical problem in physics.
- The key parameters that govern scattering and absorption of light by an sphere are

1. The wavelength λ
2. The diameter of the sphere D
3. Index of refraction of the sphere

$$x = \pi D / \lambda$$

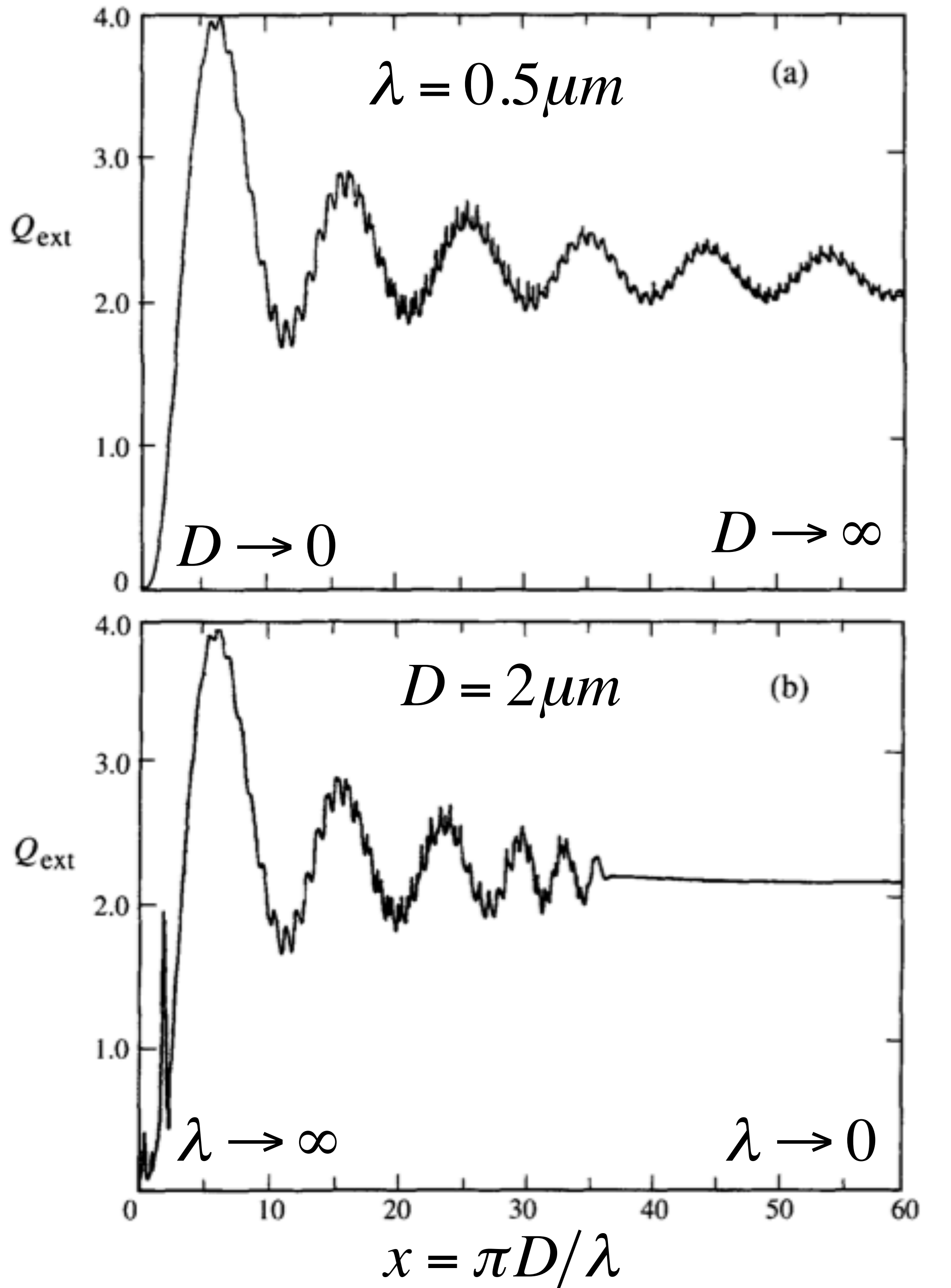
$$\tilde{n} = n + i\kappa$$

- The mathematical formalism used to solve this problem is called **Mie Theory**.

Mie Theory

Mie theory is the basis of a computational procedure to calculate the scattering and absorption of light by any sphere as a function of wavelength.

- Limiting cases:
  - **$\pi D/\lambda$ <<** 1 Rayleigh scattering
    - $Q_{scat} \sim \lambda^{-4}$ and $Q_{abs} \sim \lambda^{-1}$
  - **$\pi D/\lambda \sim$** 1 Mie scattering
    - Q_{scat} and Q_{abs} vary a lot with **x** and **ñ**
  - **$\pi D/\lambda$ >>** 1 Geometric optics
    - Reflection, refraction and difraction



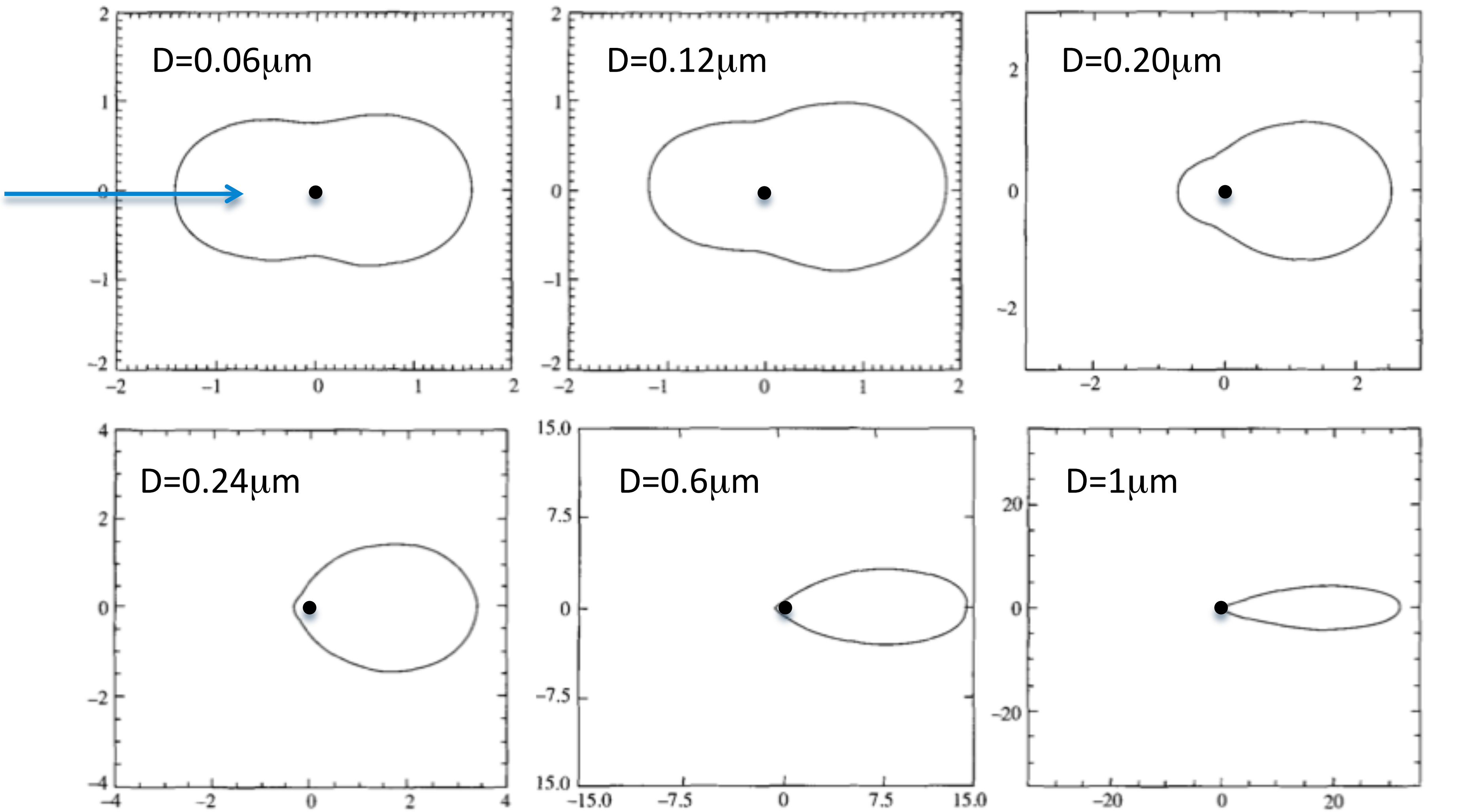
Mie Theory

Extinction efficiency of a water droplet approaches the limiting value 2, i.e. twice as much as predicted by geometric optics

Seinfeld & Pandis, cap. 15

Phase function

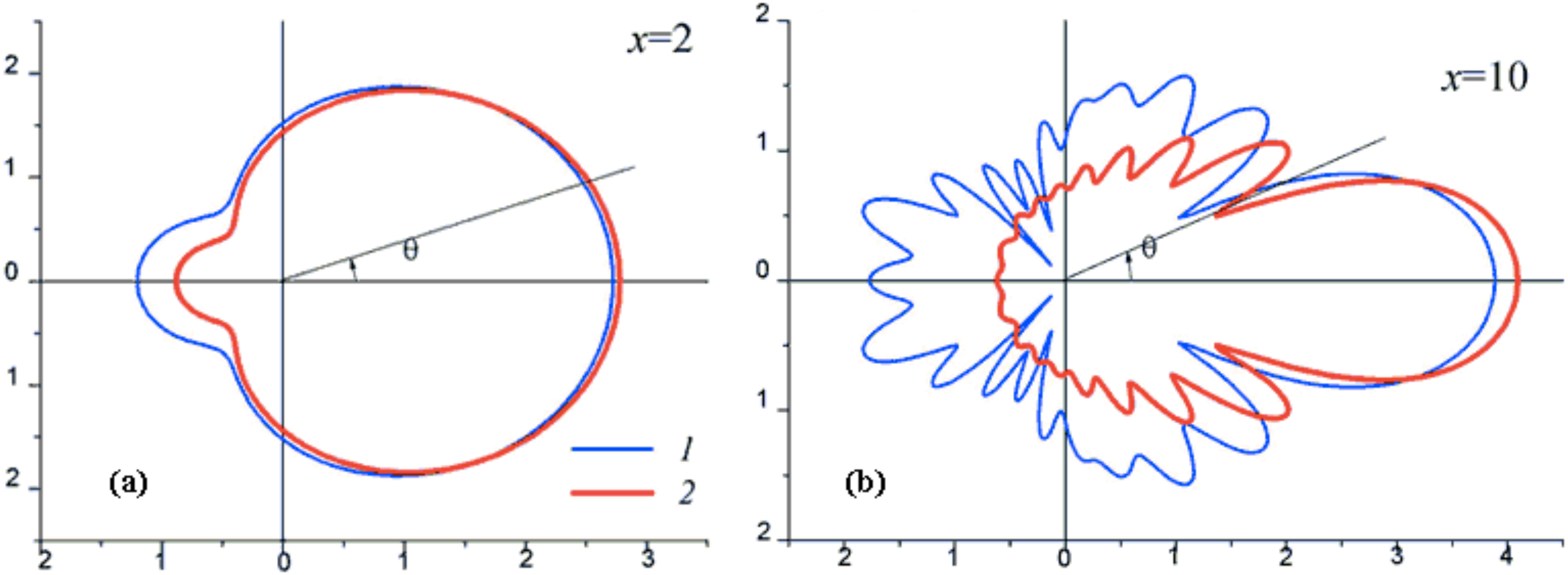
Example
$(NH_4)_2SO_4$ RH=80% λ=550nm



Seinfeld & Pandis, cap. 15

Phase function

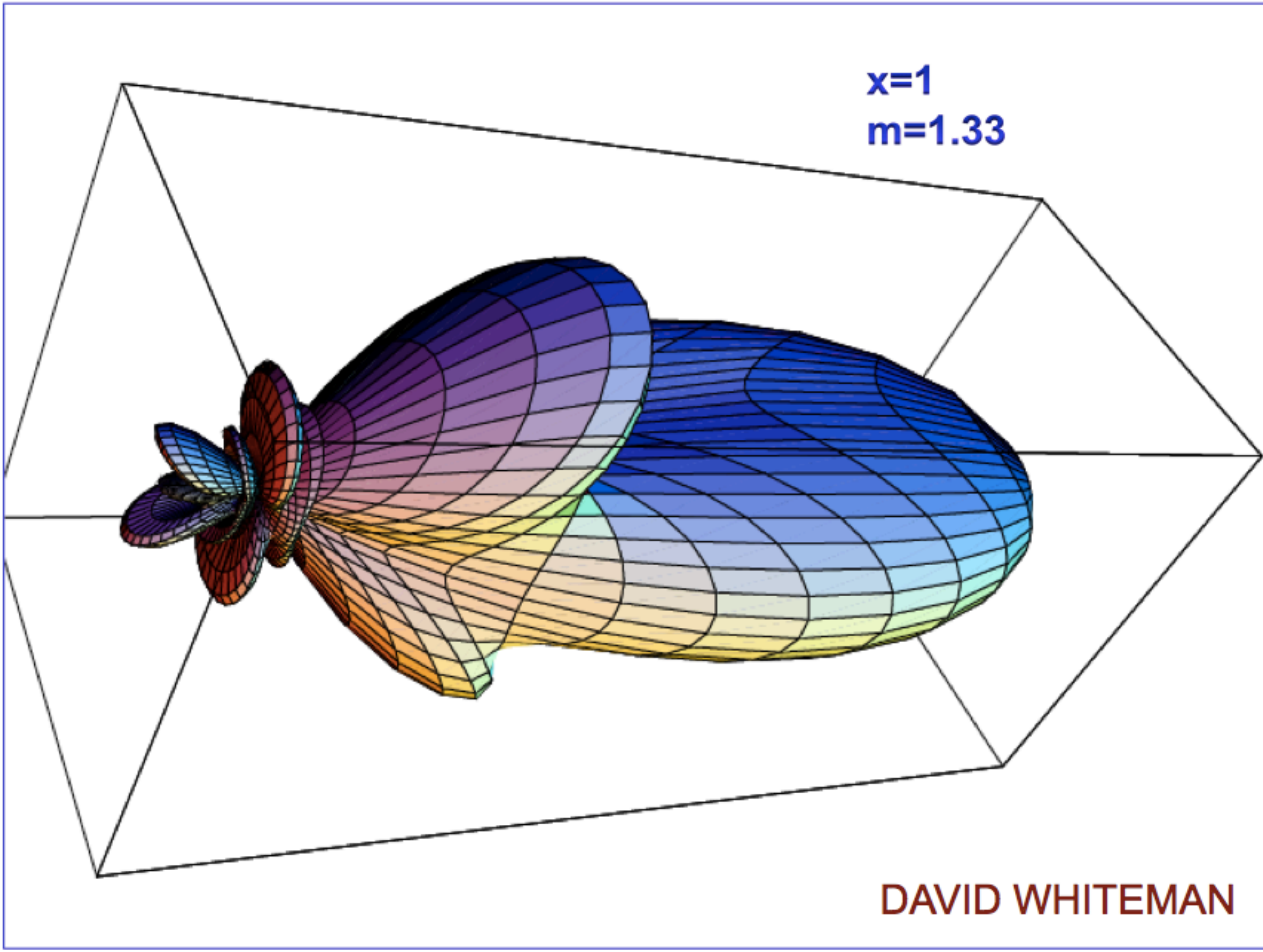
The angular distribution of light intensity scattered by a particle at a given wavelength is called the *phase function*.



ñ = 1.5 + i*0.005

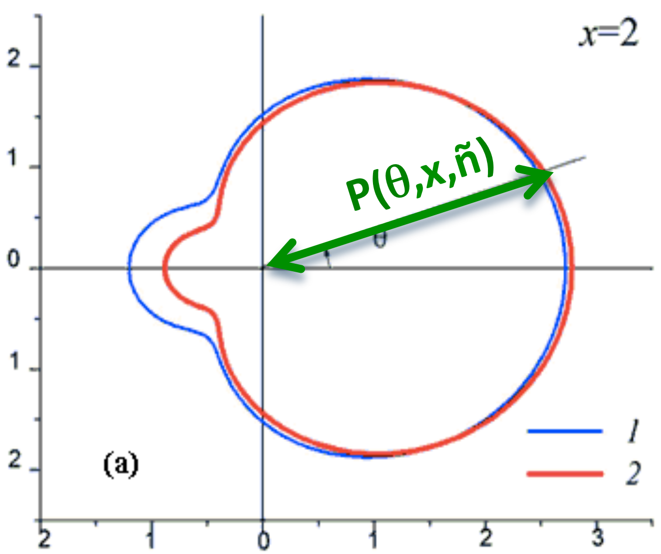
ñ = 1.5 + i*0.2

http://www.thermopedia.com/content/144/


x=1
m=1.33
DAVID WHITEMAN

Phase function

It is the scattered intensity at a particular angle θ normalized by the total scattered intensity considering all angles.



$$\int_0^{2\pi}\int_0^{\pi} P(\theta, x, \tilde{n}) \sin\theta \, d\theta \, d\varphi = 4\pi$$

The **angular volume-scattering coefficient** [m^{-1} sr^{-1}], β, is

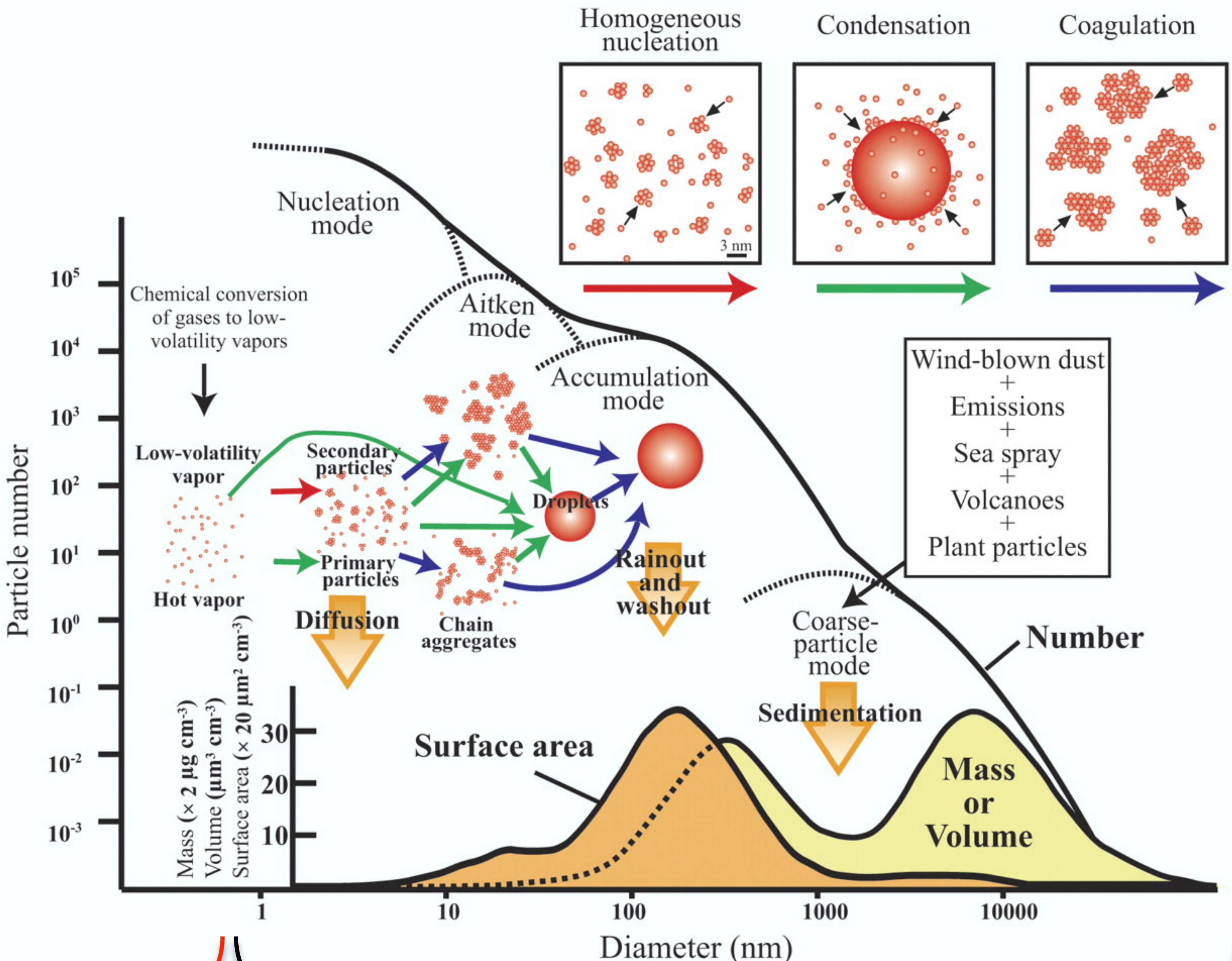
$$\beta(\theta, x, \tilde{n}, \lambda) = \alpha_{scat}(\lambda) \frac{P(\theta, x, \tilde{n})}{4\pi}$$

Asymmetry parameter

- The asymmetry parameter **g** is defined as the intensity-weighted average of the cosine of the scattering angle:

$$g = \frac{1}{2}\frac{\int_0^\pi \cos\theta \; F(\theta)\sin\theta \, d\theta}{\int_0^\pi F(\theta)\sin\theta \, d\theta} = \frac{1}{2}\int_0^\pi \cos\theta \; P(\theta)\sin\theta \, d\theta$$

- The factor of ½ ensures that **g = 1** for light scattered totally at **θ=0°** (forward) and **g = -1** for light scattered completely at **θ=180°** (backward).



gases

aerosols

Points to remember #2

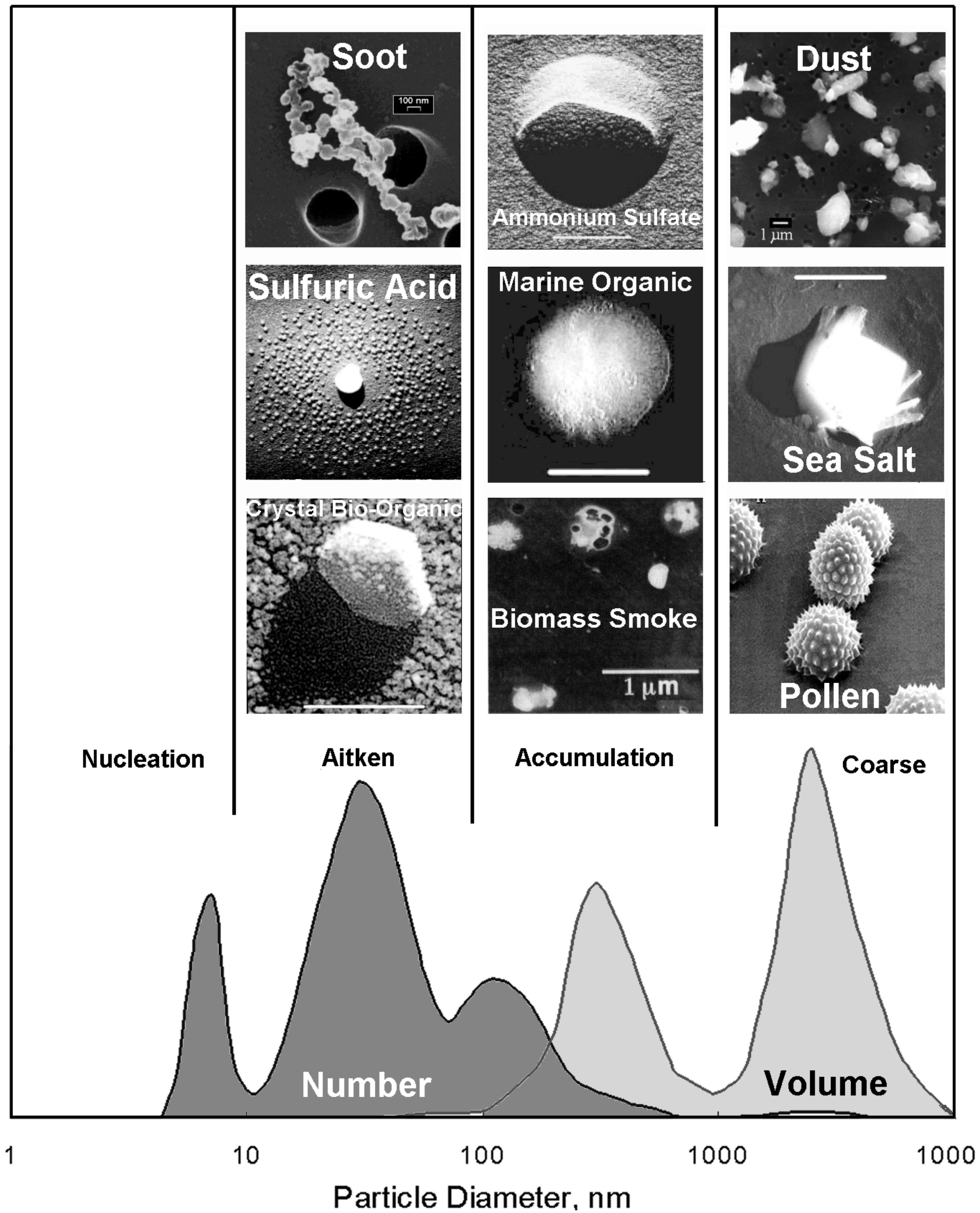
- Aerosol and molecule interaction with the radiation depends on:
  - Size

$$x = \pi D / \lambda$$

  - Shape

  - Surface properties

$$\tilde{n} = n + i\kappa$$



Points to remember #3

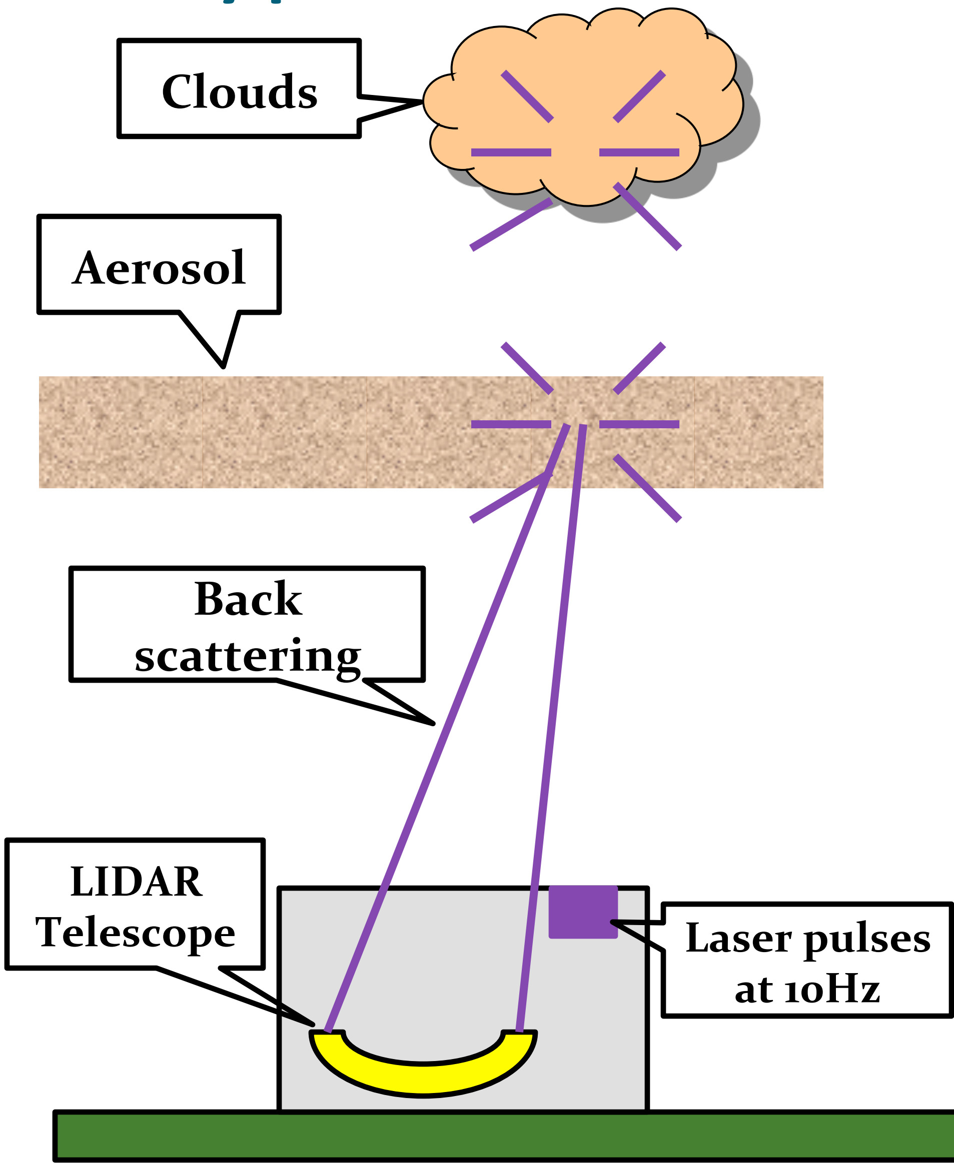
- The atmosphere has
  - Molecules (~ 0.1 to 0.5 nm)
  - Aerosol particles (10nm to 1μm)
- Solar radiation ~ 0.1 to 4μm, hence 2 types of scattering
  - **πD/λ <<** 1 Rayleigh scattering - **Molecules**
  - **πD/λ ~** 1 Mie scattering - **Aerosols**
- Total extinction is given by

$$\alpha_{ext} = \alpha_{abs} + \alpha_{scat} \qquad \begin{cases} \alpha_{abs} = \alpha_{abs,g} + \alpha_{abs,p} \\ \alpha_{scat} = \alpha_{scat,g} + \alpha_{scat,p} \end{cases}$$

Outline

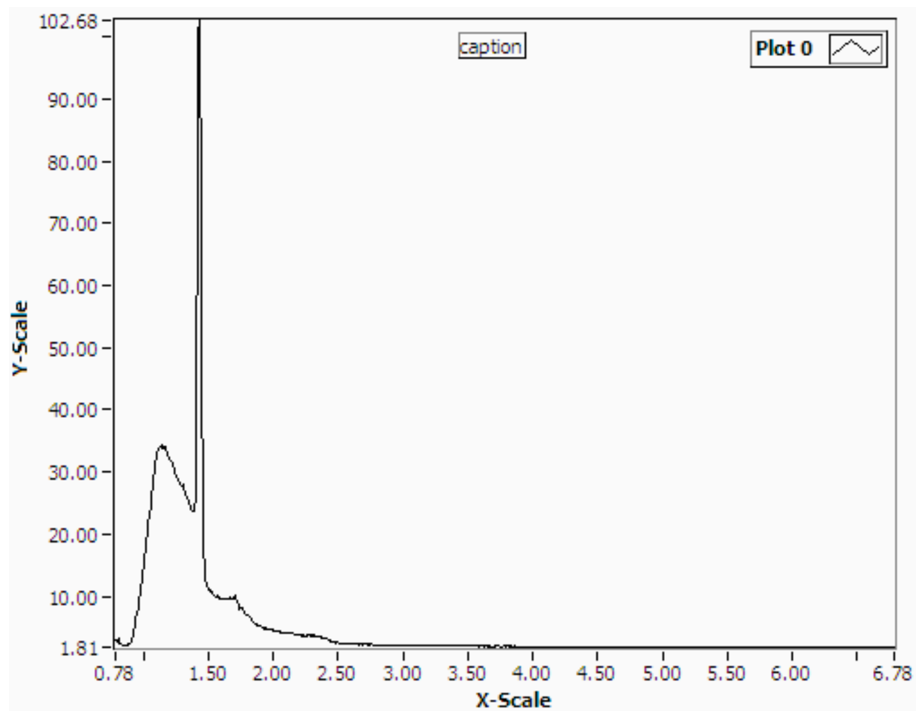
- Radiative transfer in the atmosphere
  - Beer-Bouguer-Lambert law
  - Mie Scattering
- **Typical LIDAR setup**
  - **Detector**
  - **Overlap**
- Lidar Equation
  - Molecular atmosphere
  - Klett-Fernald-Sasano
- The Lalinet (tentative) algorithm

A typical Lidar



Speed of light is **3 x 10^{8} m/s** and we measure at **20Mhz**, hence **vertical resolution is 7.5m**

We measure light intensity x time



Typical setup

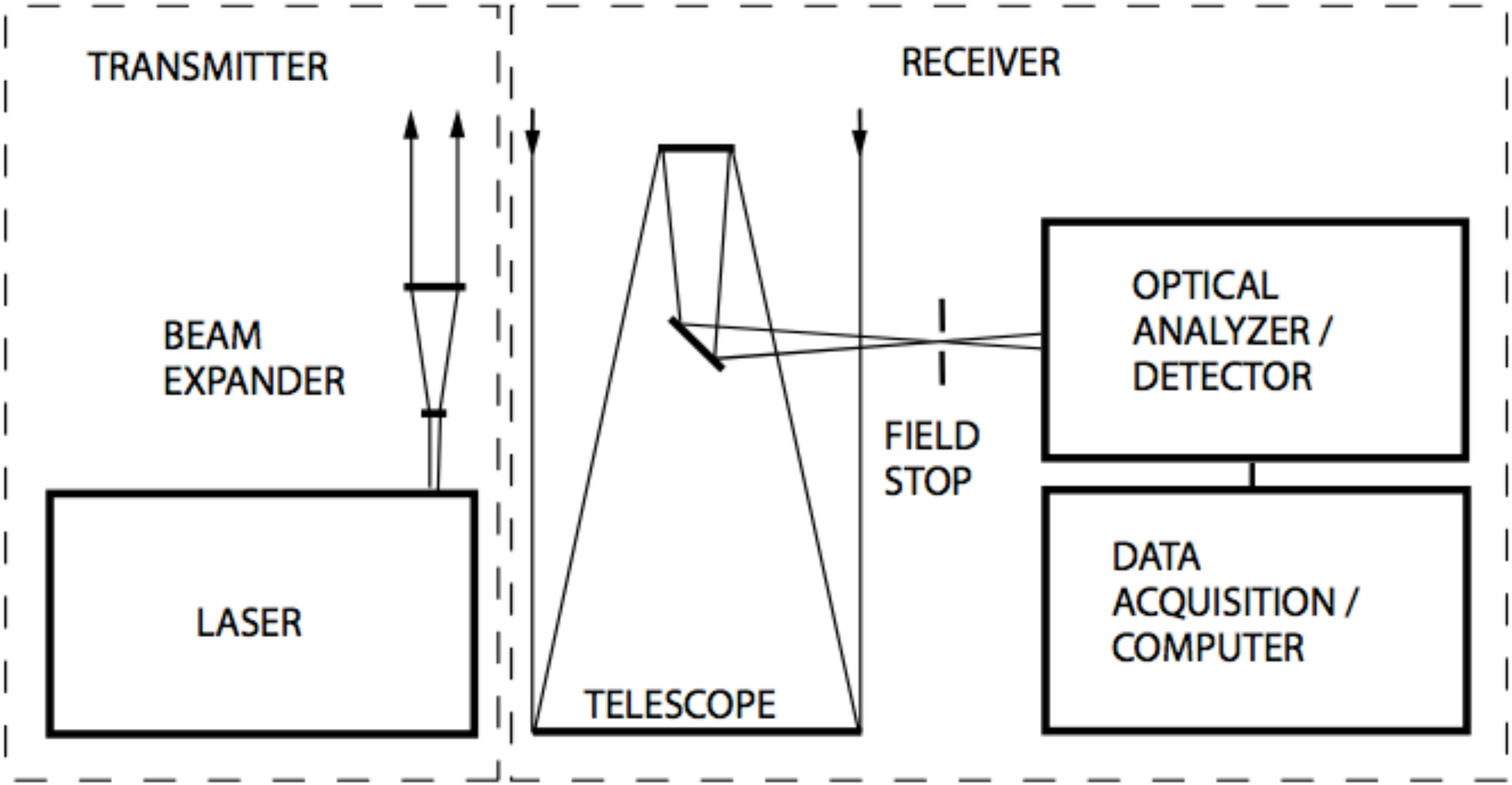
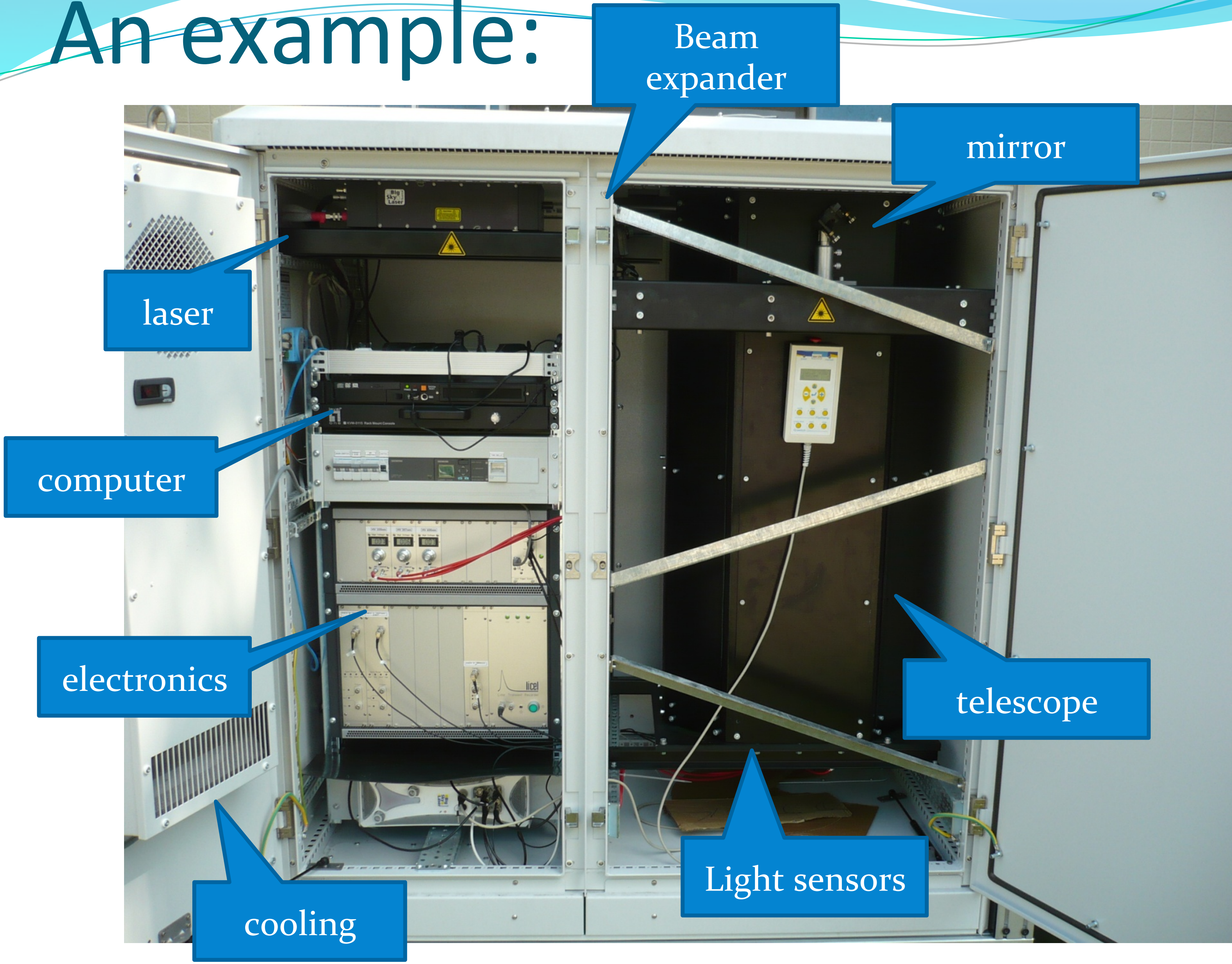


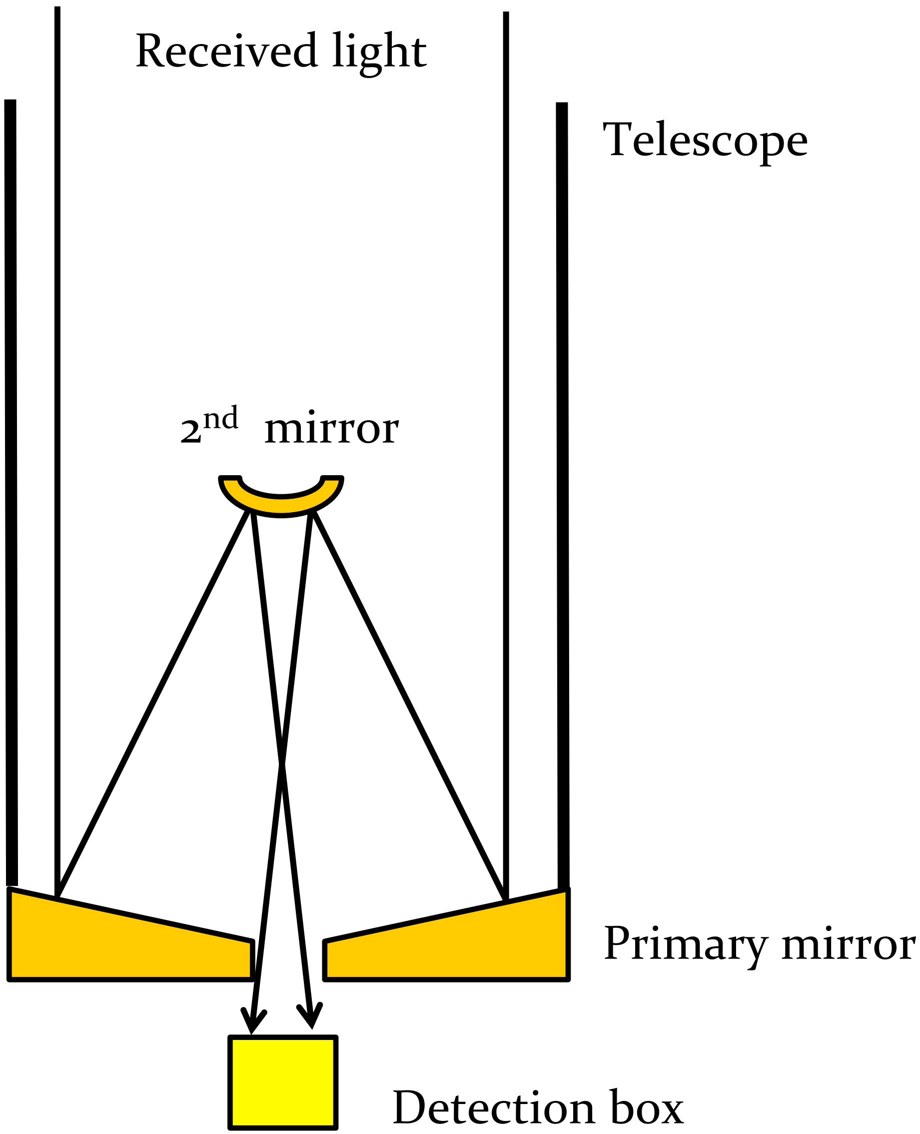
Fig. 1.1. Principle setup of a lidar system.

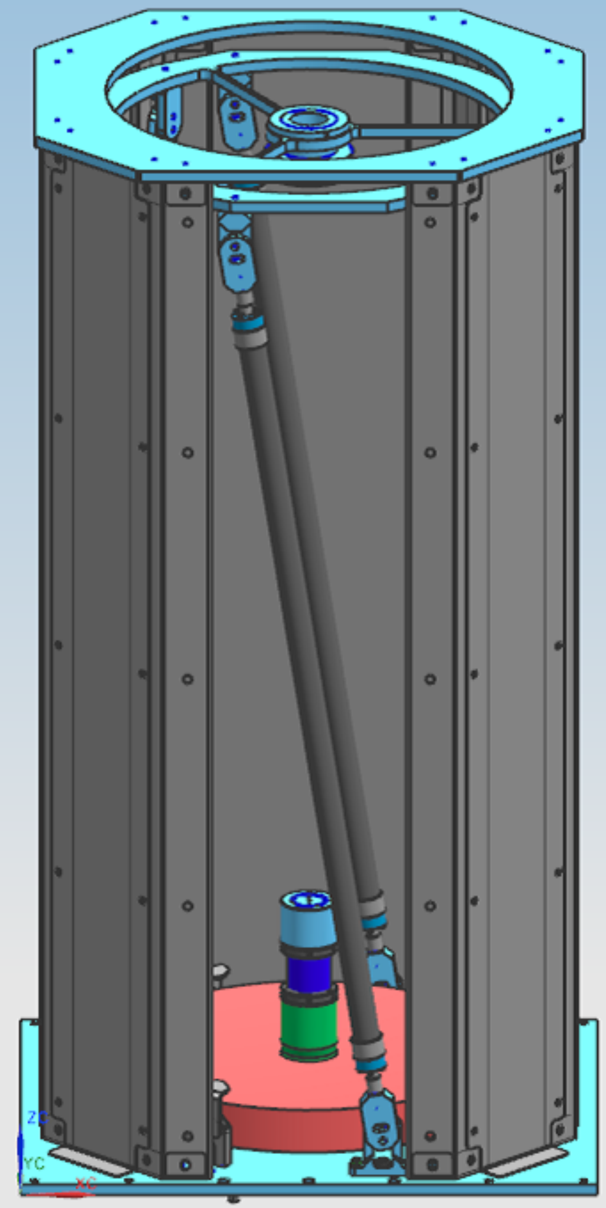
Weitkamp, chap 1

An example:



Beam expander
mirror
laser
computer
electronics
telescope
Light sensors
cooling

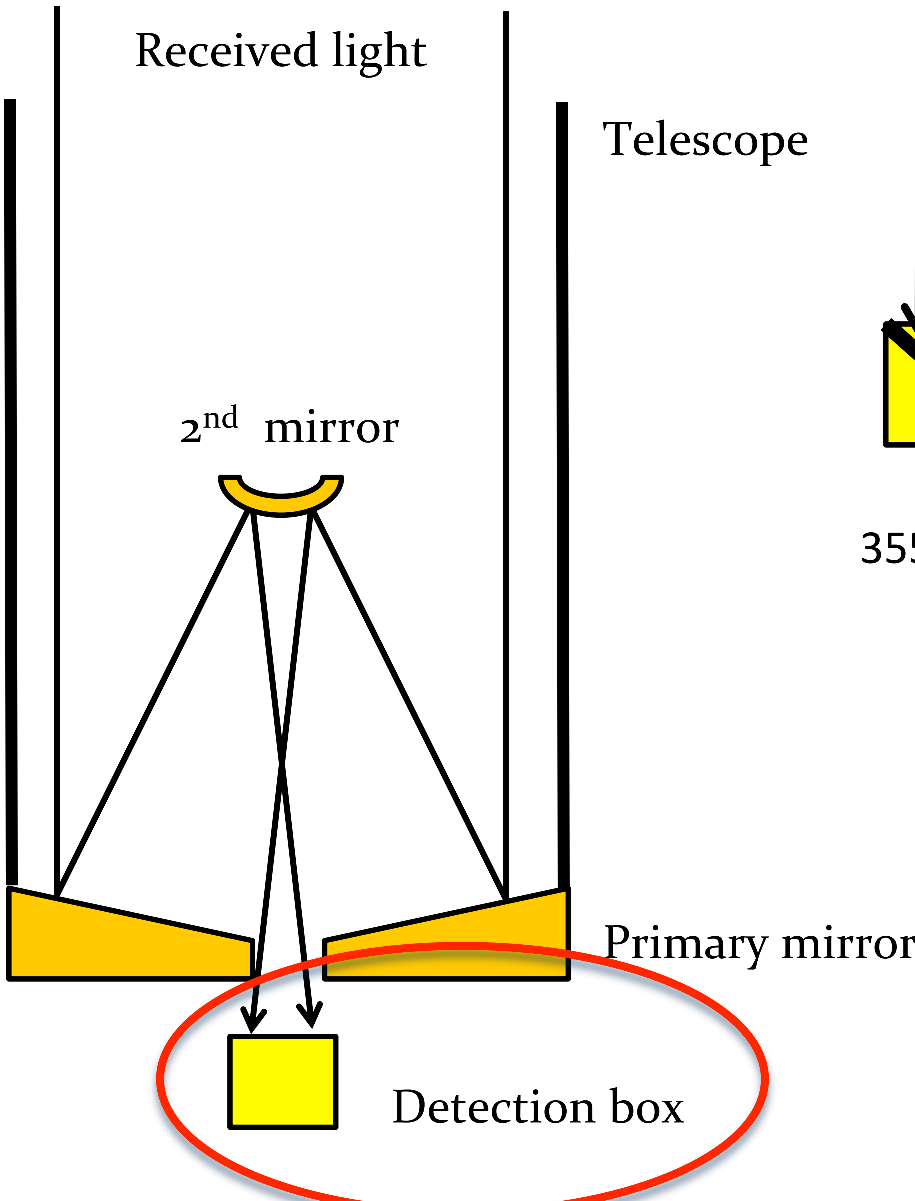

Received light
Telescope
2nd mirror
Primary mirror
Detection box

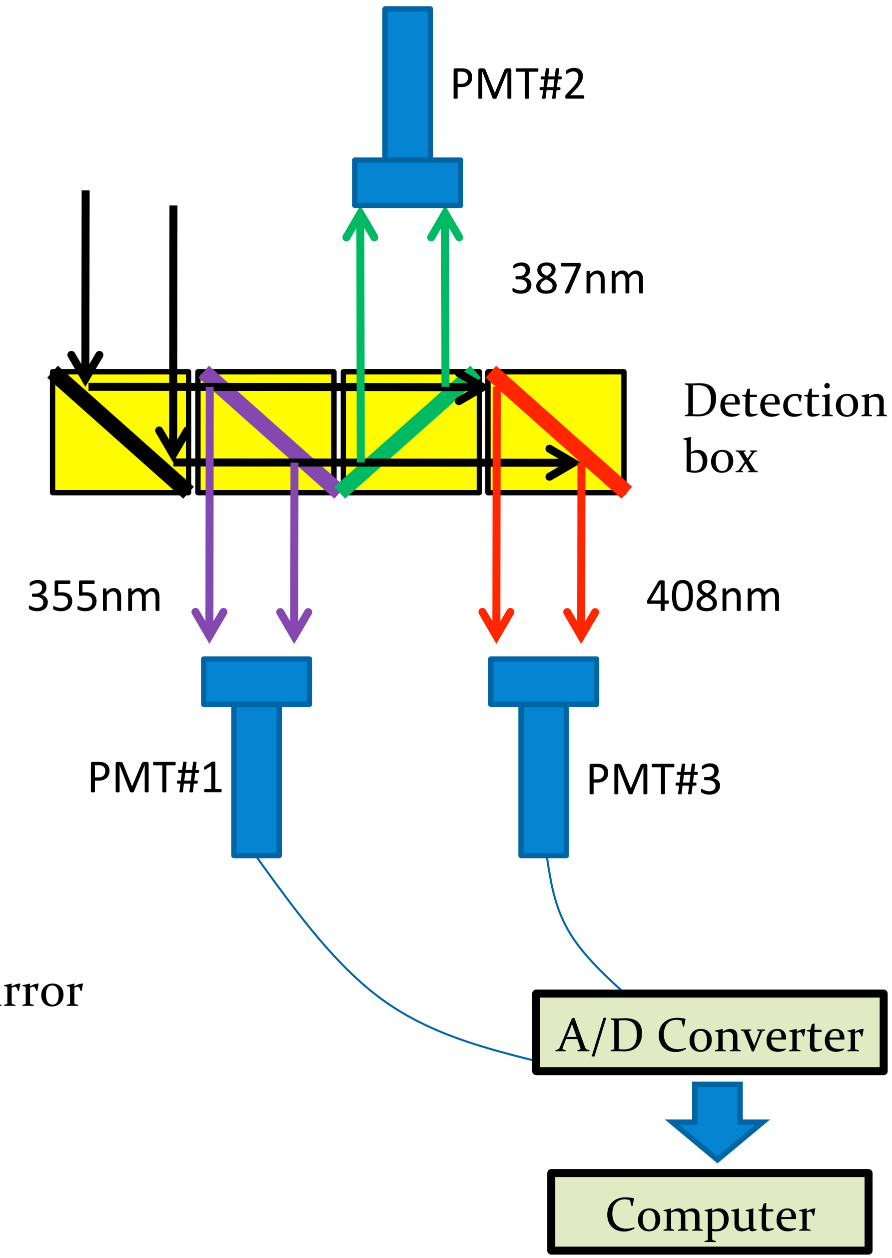



Secondary mirror


Primary mirror

Optical layout - Detection

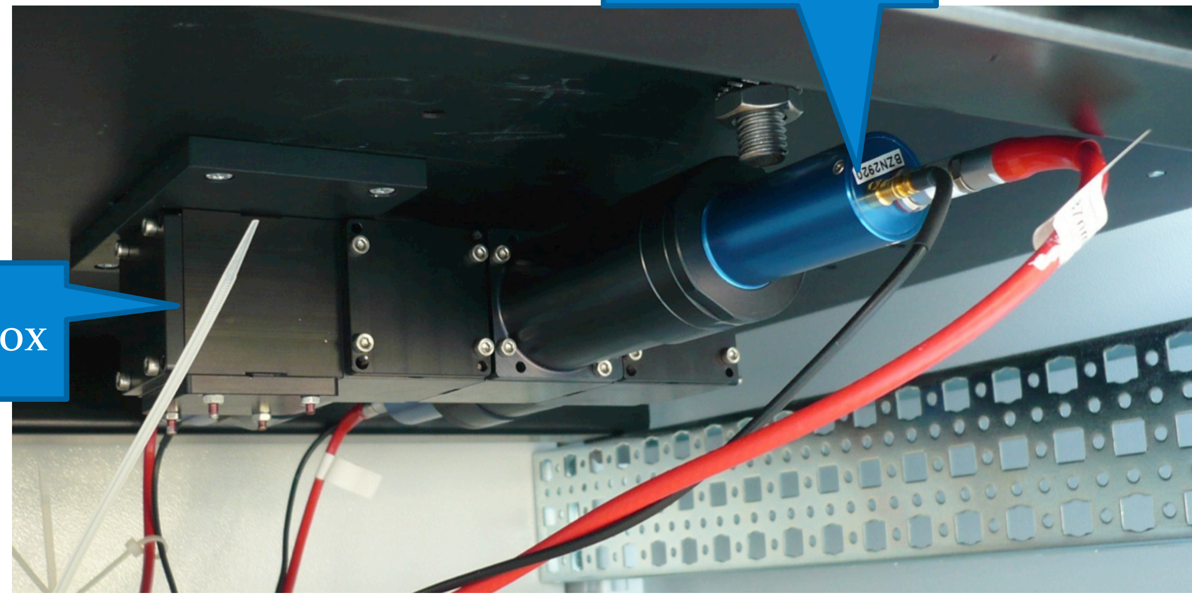




Optical layout – Detection

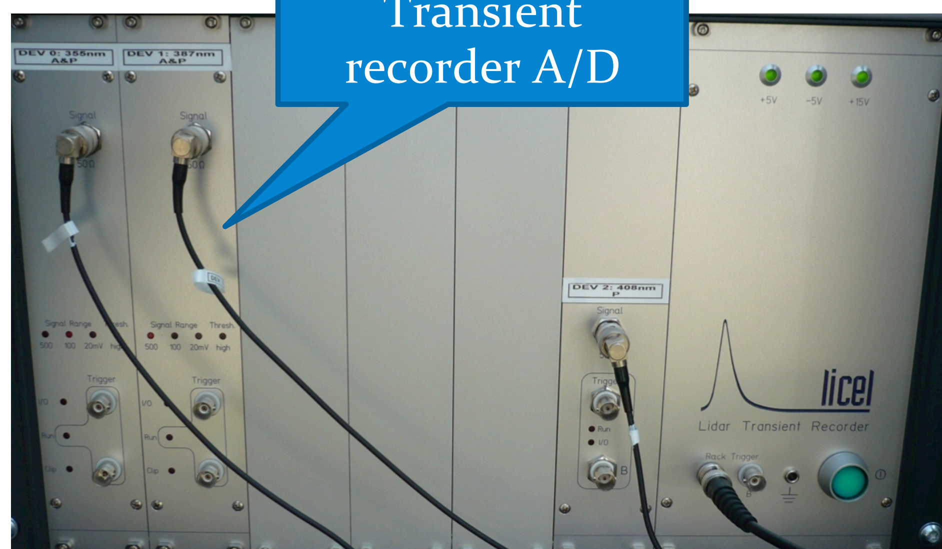
PMT

Detection box

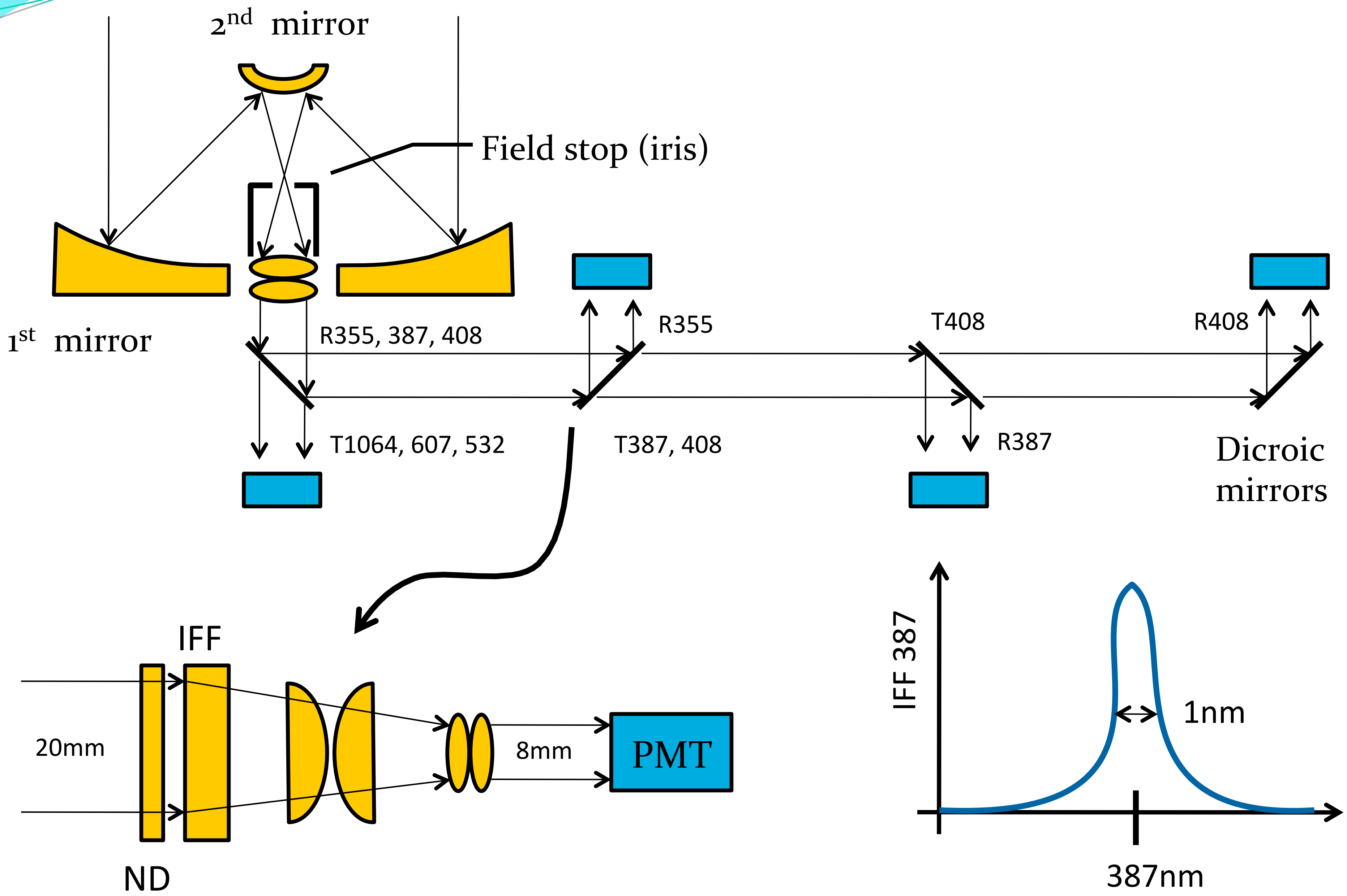


HV control

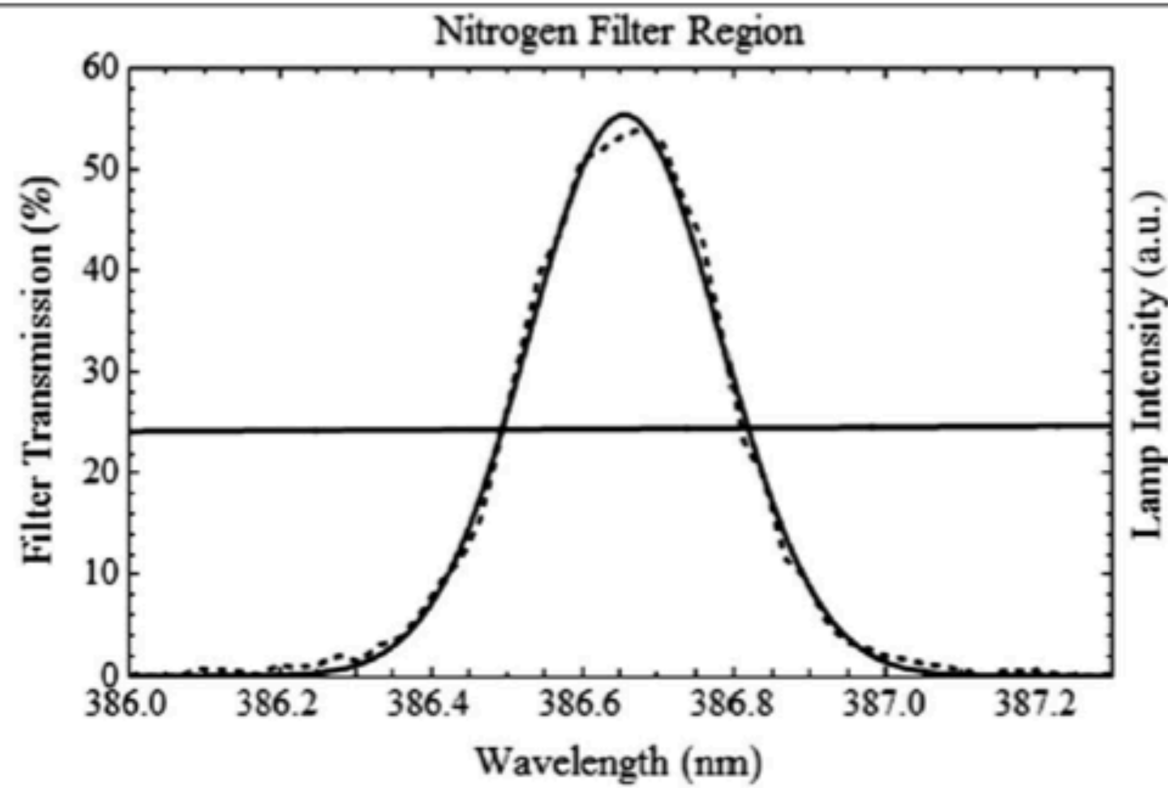
Transient recorder A/D



Optical layout - Detection



SPECTRAL RESPONSE



$$\mathcal{F}_{Gauss} = C_1 Exp\left[-\frac{1}{2}\left(2\sqrt{2\log_e[2]}\ \frac{x-C_2}{C_3}\right)^2\right]$$

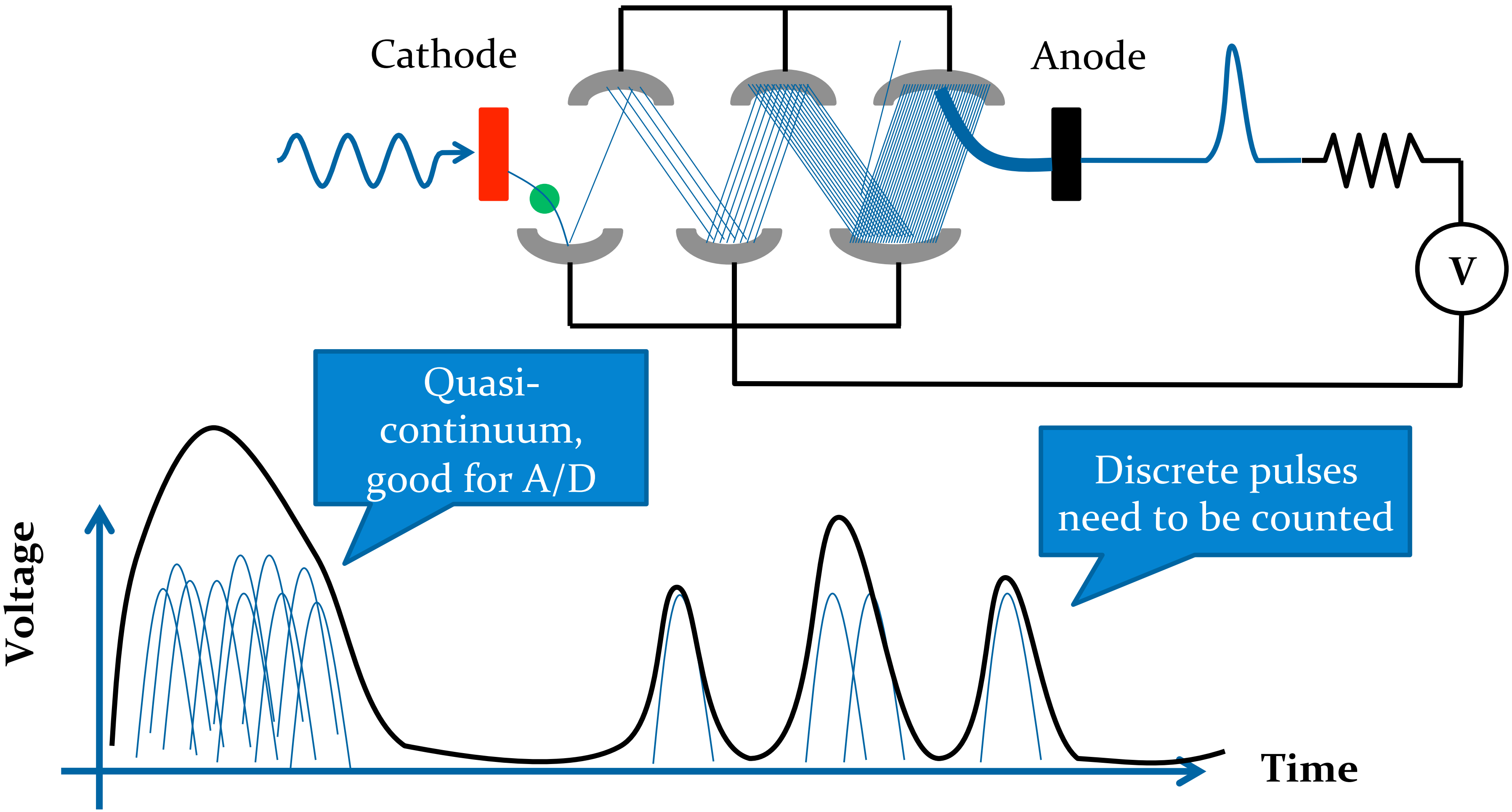
Filter	λ_o (nm)	FWHM (nm)	Amplitude (%)
N	386.67	0.30	55.41
H	407.51	0.24	48.53

VENABLE et al.

Landulfo, 2012

PMT

1 photon => 10^7 photo electrons



Signals overlap and give
large measurable voltage

Pulses too far way

Efficiency < 30%
Hamamatsu, TECH 2001

Efficiency is not uniform over Anode

Simeonov et al, Ap. Opt. 1999

Figure 12(a): Transmission Mode Photocathodes

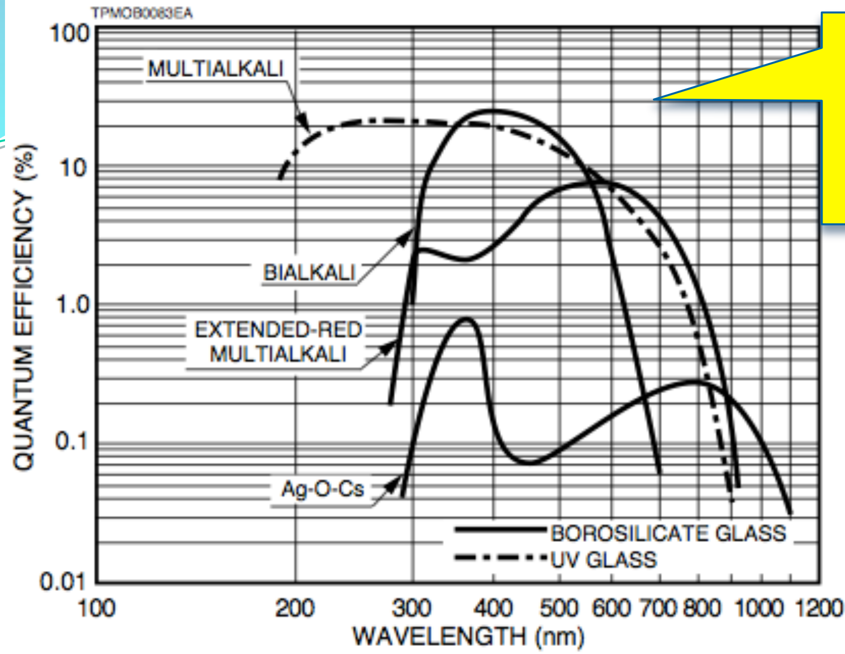
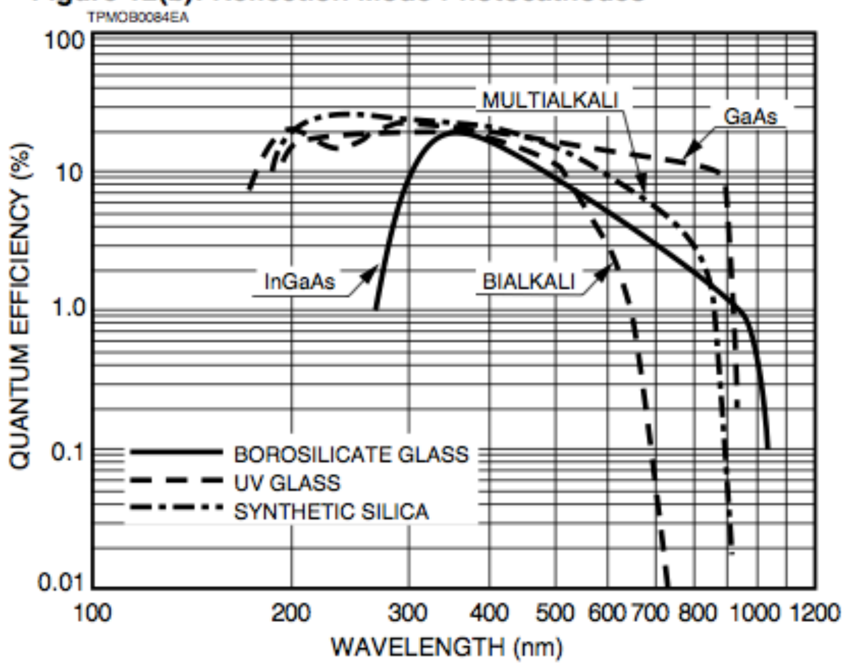


Figure 12(b): Reflection Mode Photocathodes



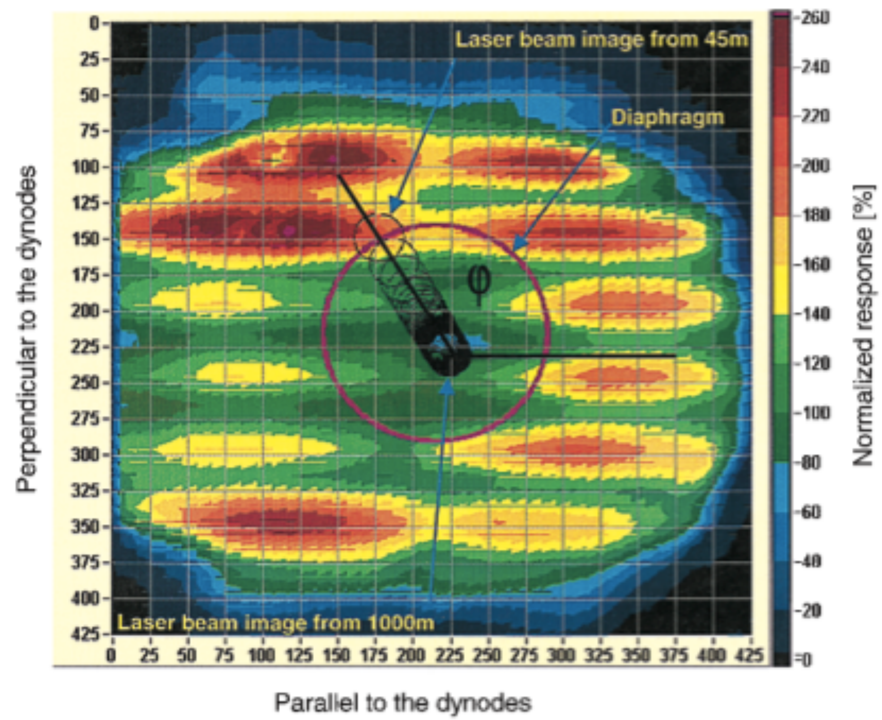
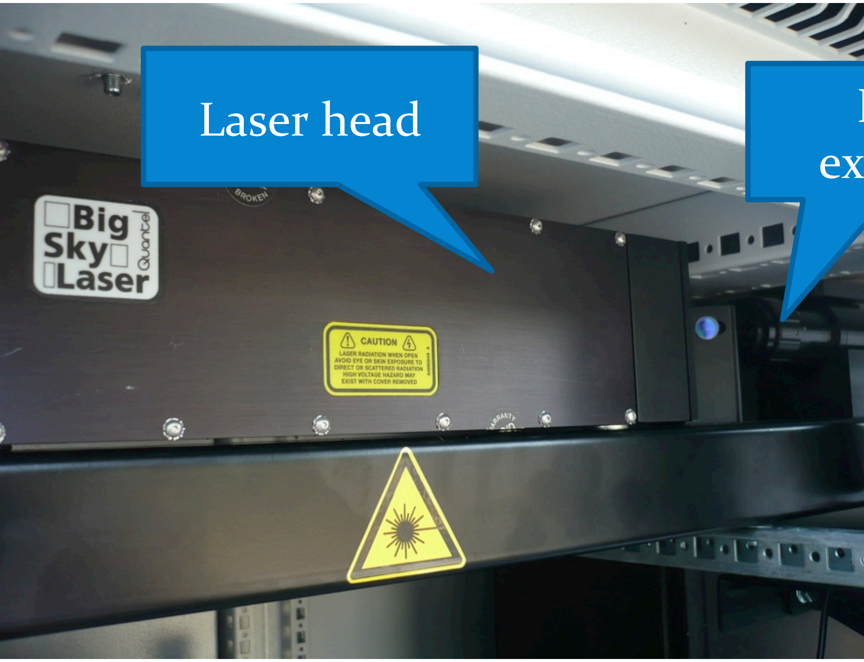
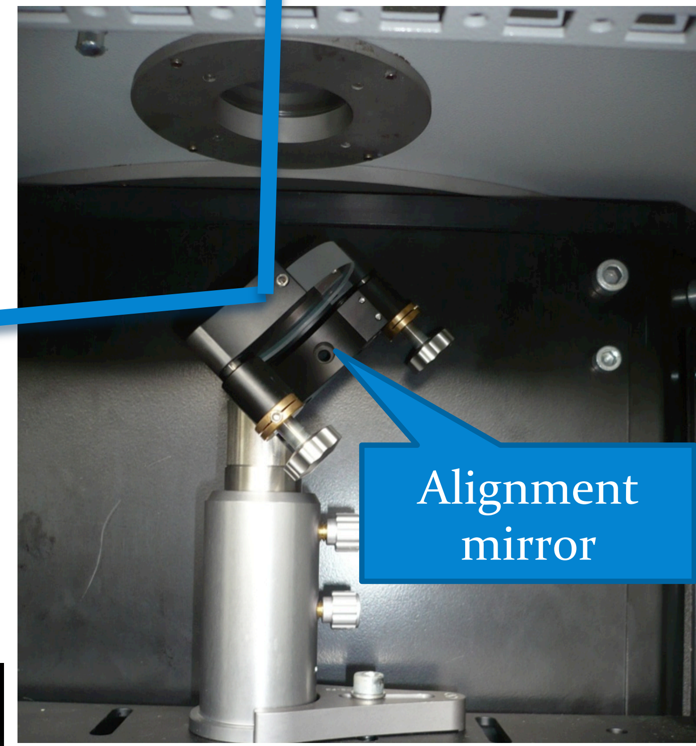


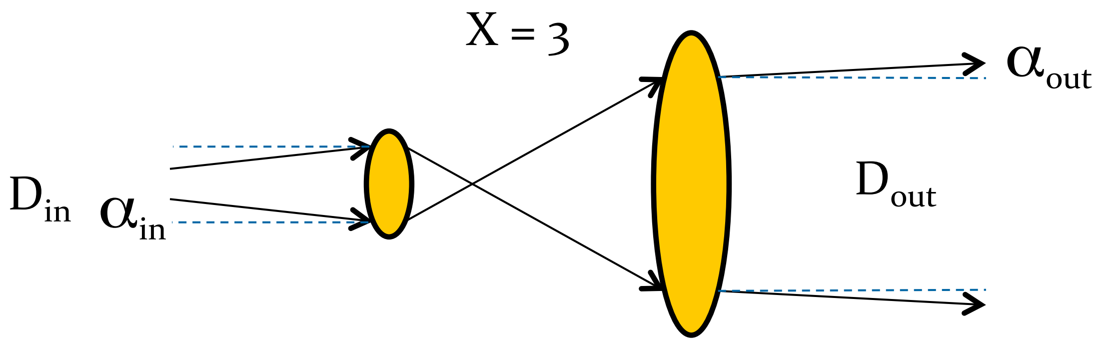
Fig. 3. Anode spatial uniformity from Fig. 1 with resolution enhanced ten times by two-dimensional interpolation (20 × 20 μm). A model image of the probing laser beam with a range resolution of 15 m (sequence of black circles) and the receiving telescope field stop (violet circle).

Optical layout - Emission



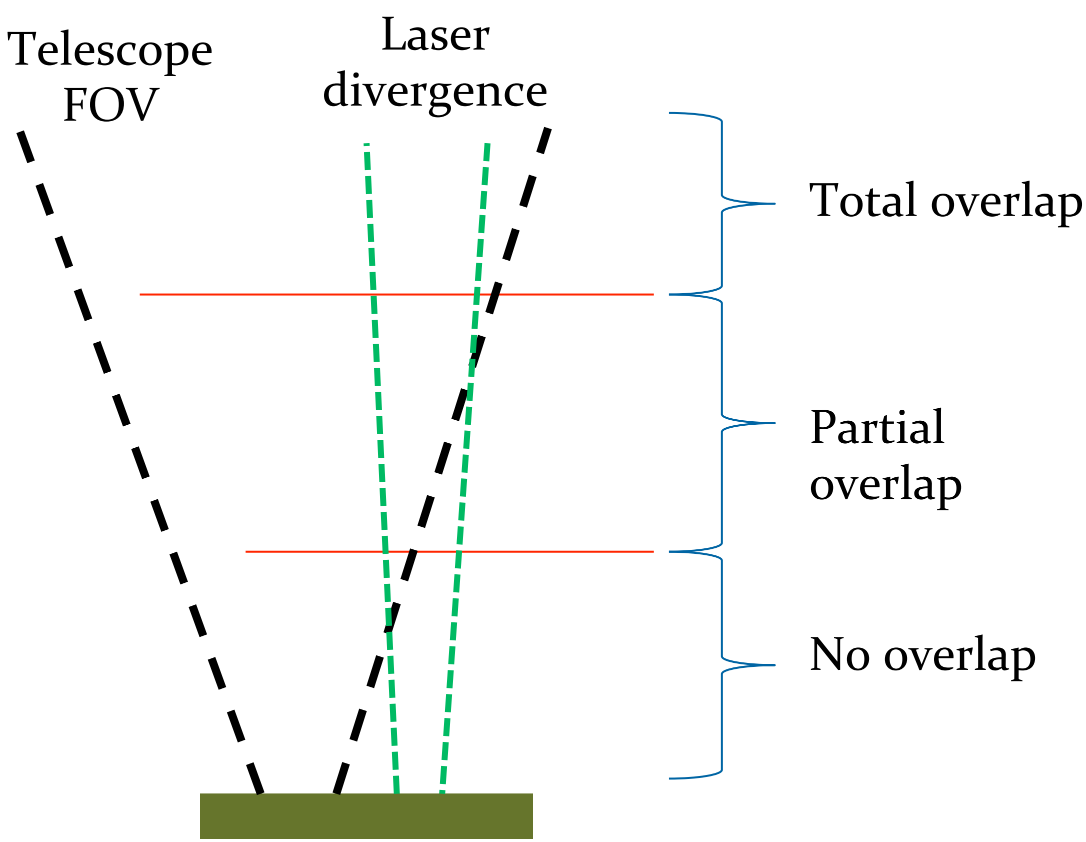


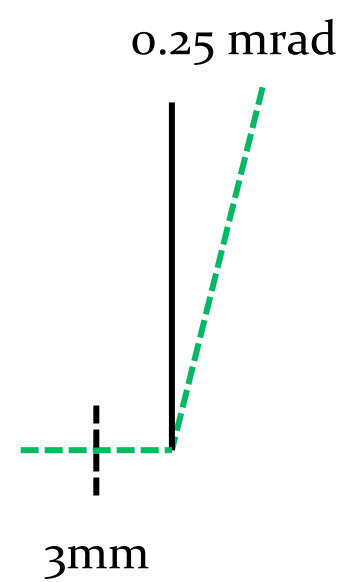
Beam div x FOV tel



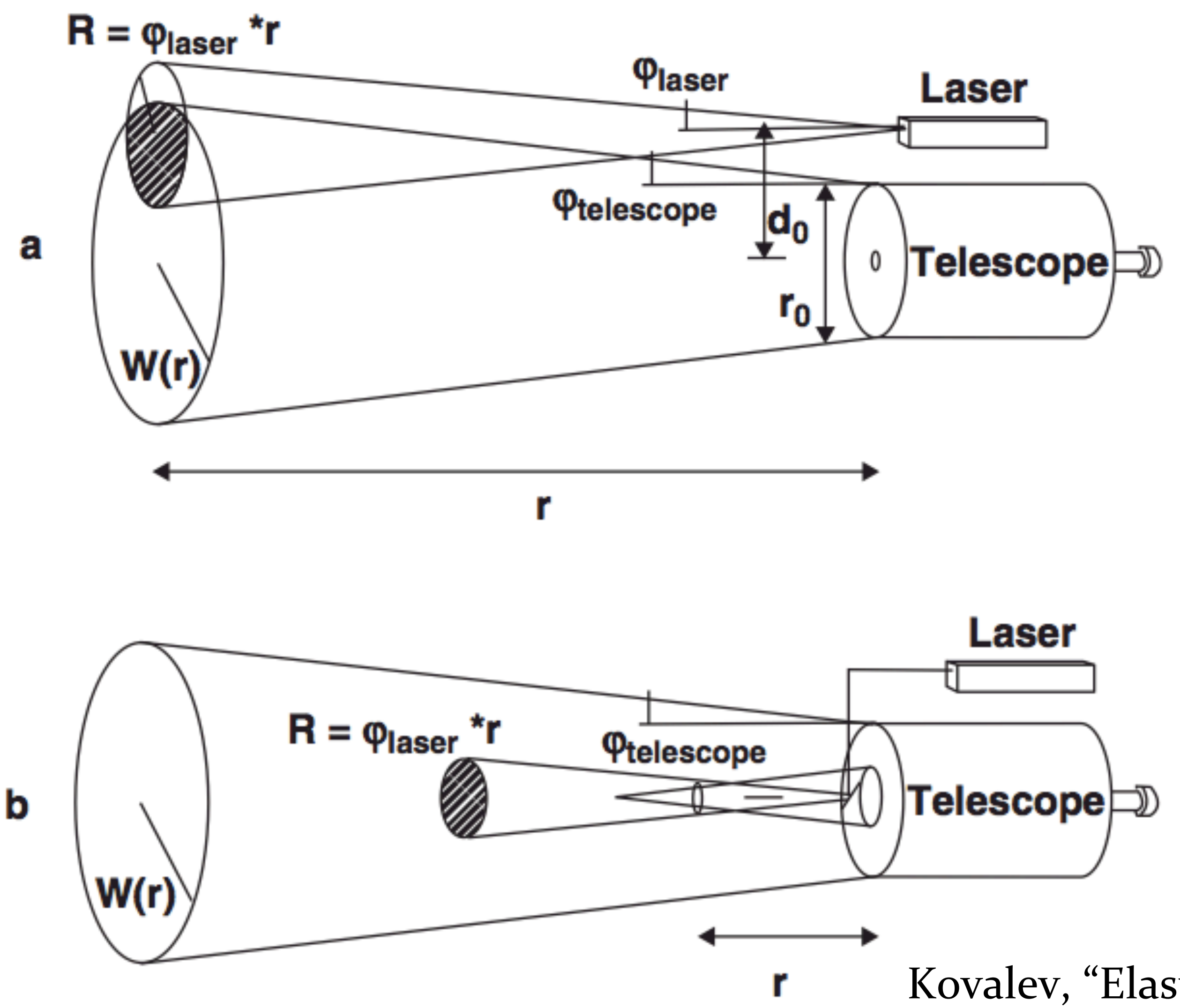
$$D_{out} = x \cdot D_{in} = 3 \cdot 1mm = 3mm$$

$$\alpha_{out} = \frac{\alpha_{in}}{x} = \frac{0.75mrad}{3} = 0.25mrad$$





Mono- vs Bi-axial systems



Kovalev, “Elastic Lidar”, chap. 3

Overlap function

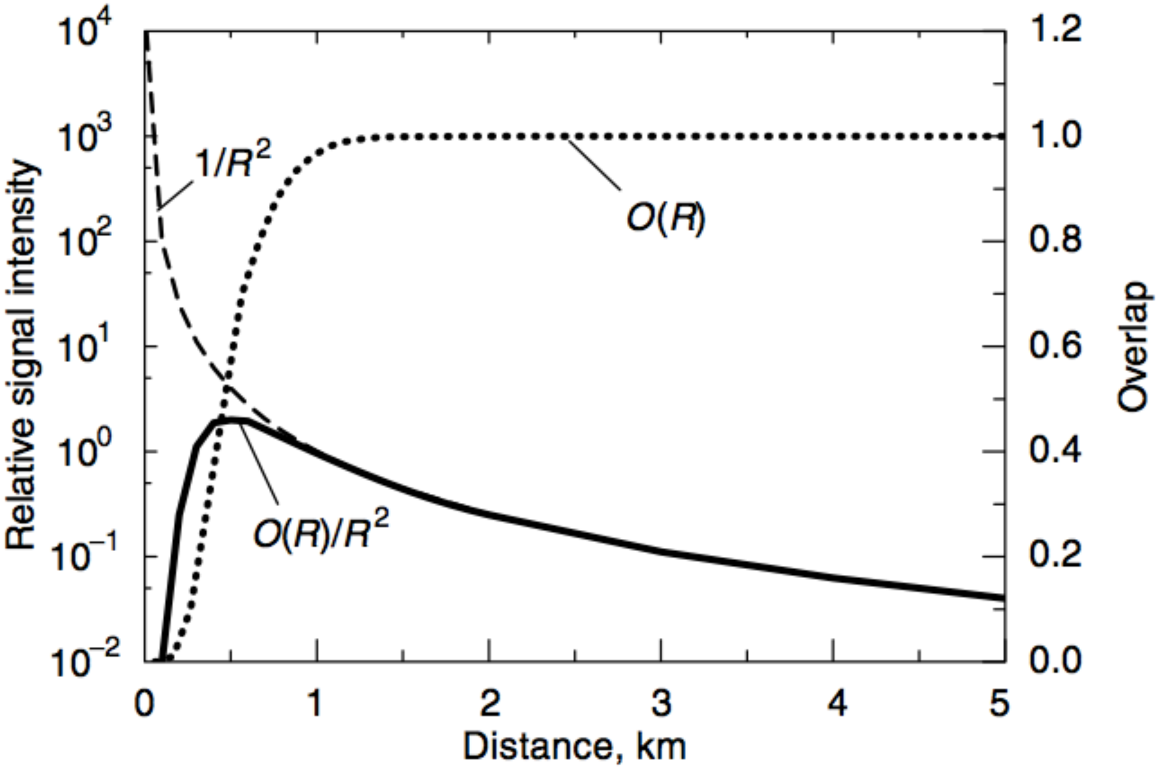
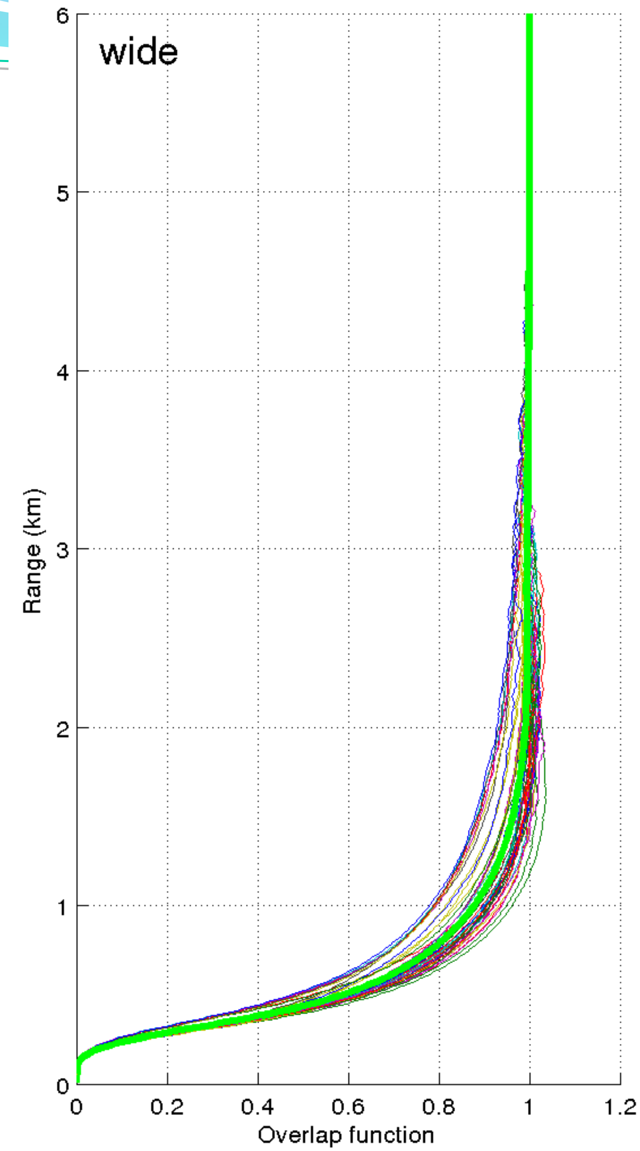


Fig. 1.3. Influence of the overlap function on the signal dynamics.



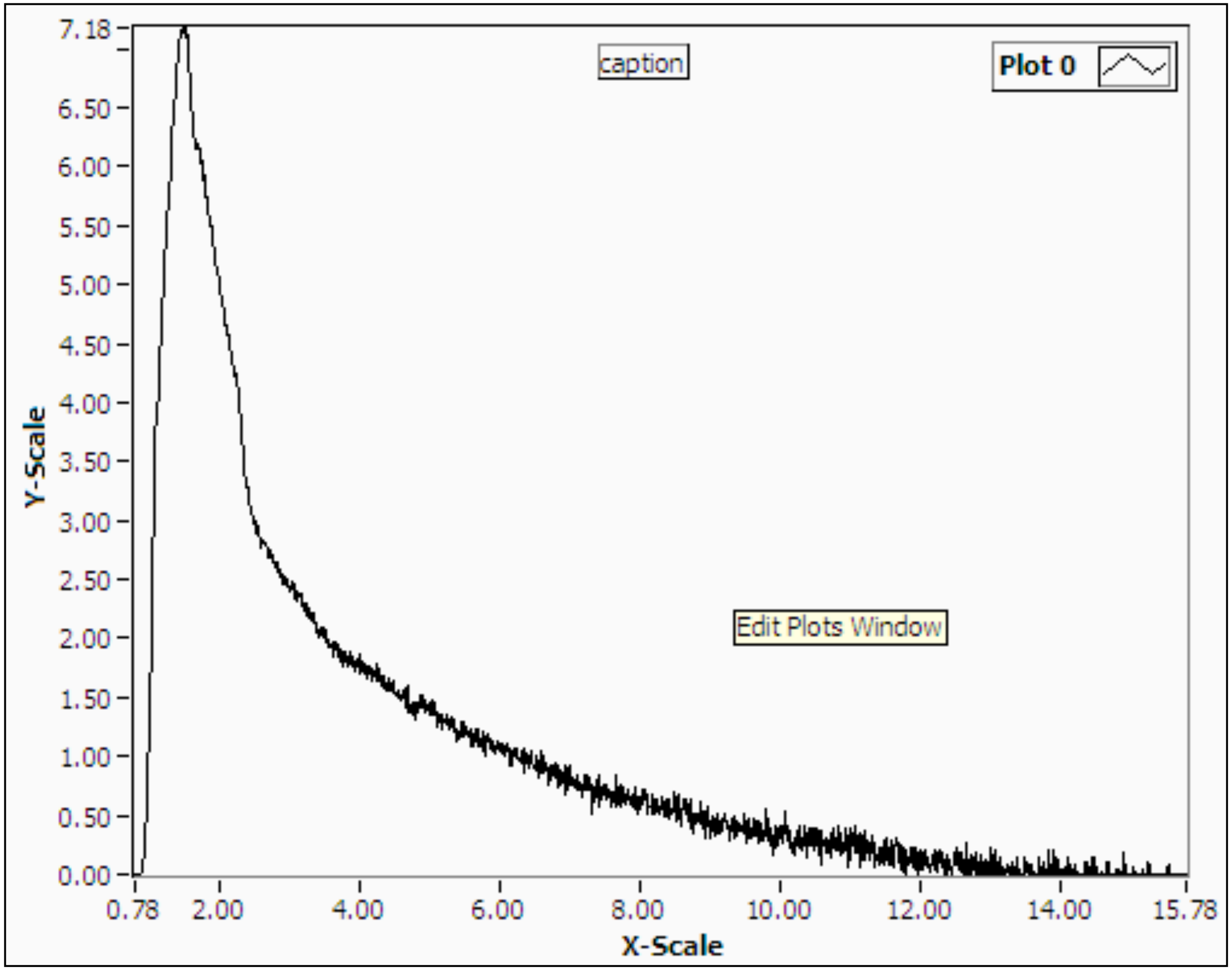
Weitkamp, chap 1

Outline

- Radiative transfer in the atmosphere
  - Beer-Bouguer-Lambert law
  - Mie Scattering
- Typical LIDAR setup
  - Detector
  - Overlap
- **Lidar Equation**
  - **Molecular atmosphere**
  - **Klett-Fernald-Sasano**
- The Lalinet (tentative) algorithm

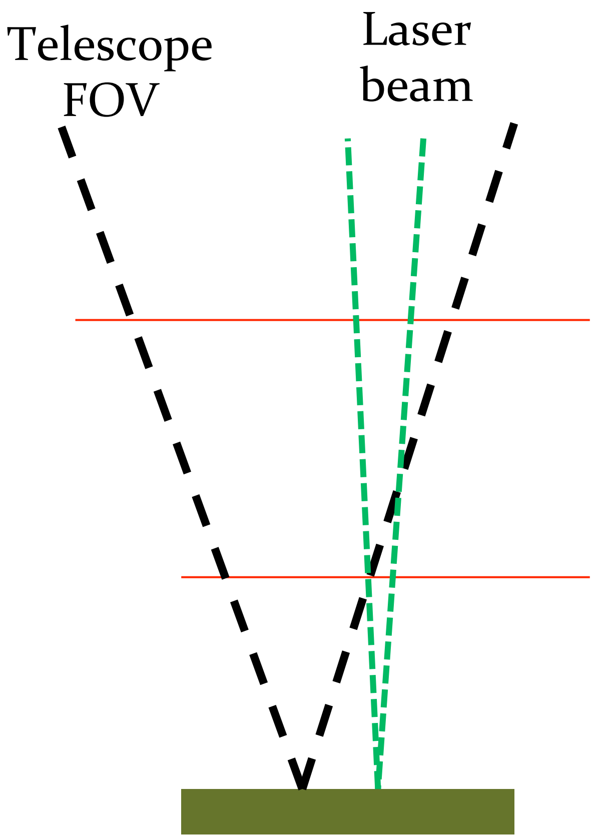
How can we describe this signal?

How does the signal vary with time (or height) ?



The simplest lidar equation

$$P(r) = K \cdot G(r) \cdot B(r) \cdot T(r)$$



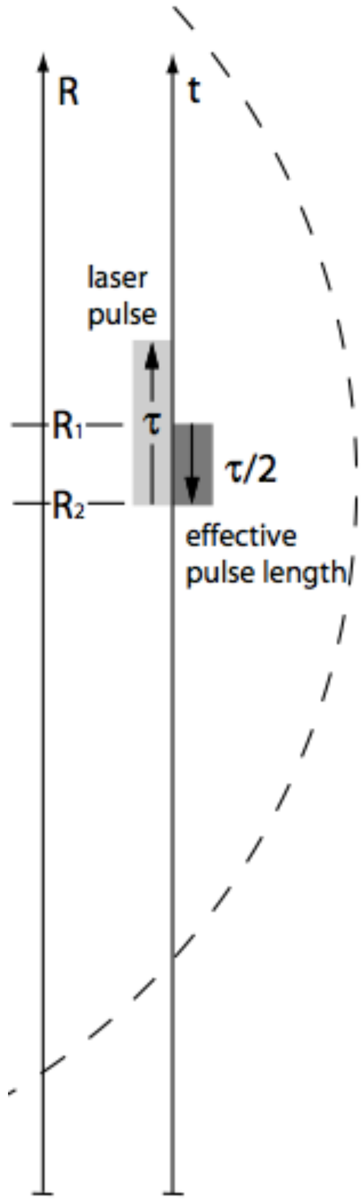
- K = System performance
- G(r) = Change of geometry with range r
- B(r) = Fraction of light scattered towards the telescope
- T(r) = Atmospheric transmission

System performance

- Number of photons emitted $= P_0$
- Detection efficiency $= \eta(\lambda)$
- Effective pulse length $= \dfrac{c\tau}{2}$

Photons need to go out, scatter, and come back…

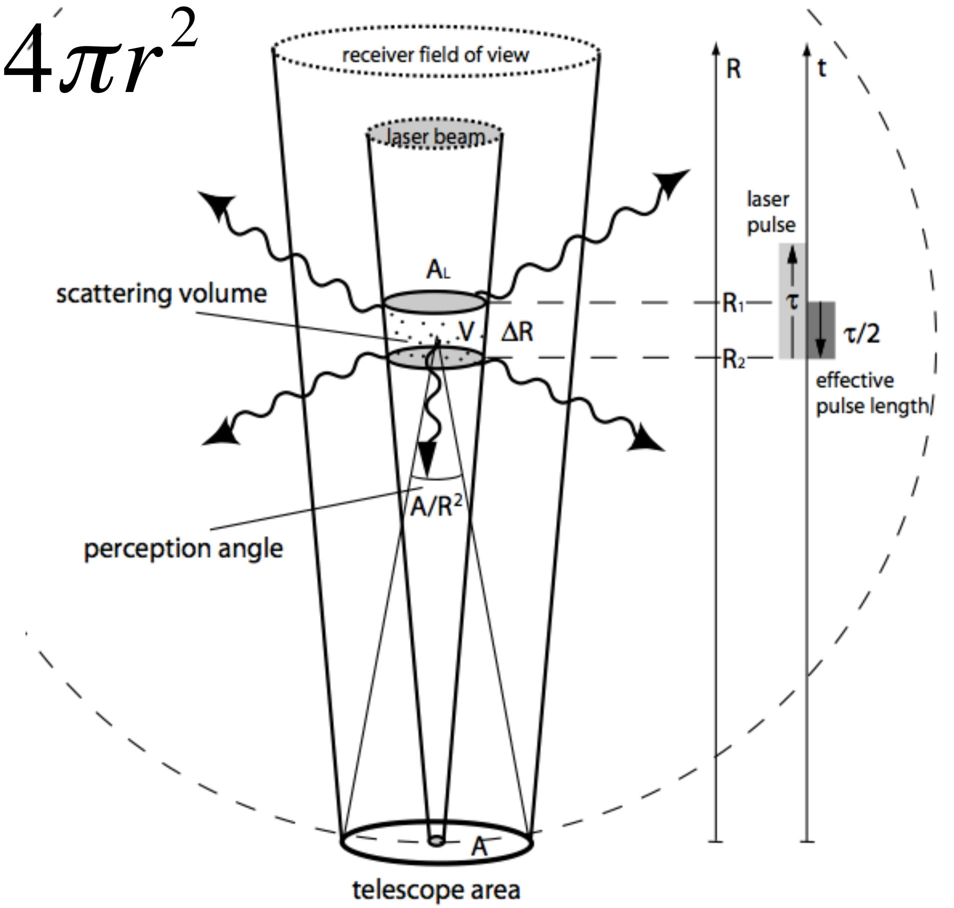
$$z = c\frac{t - t_0}{2} = range$$



Change of Geometry

Note that it implies isotropic scattering.

- Solid angle subtended $= \dfrac{A}{4\pi r^2}$
- Overlap factor $= O(r)$



Weitkamp, chap 1

Back-scatter coefficient

- For any angle θ, we had:

$$\beta(\theta, x(r), \tilde{n}(r), \lambda) = \alpha_{scat}(\lambda)\frac{P(\theta, x(r), \tilde{n}(r))}{4\pi}$$

- Telescope is small, **r>>1** and **θ~π** (backscatter), and for isotropic scattering (**P=1**), the total scattering is then:

$$4\pi\beta(\pi, r, \lambda) = N(r)\sigma_{scat}(\lambda)$$

Transmission Term

- As the laser pulse travels two times the distance from the Lidar to range **r**, then the transmission term is simply:

$$T(r,\lambda) = \exp\left[-2\int_0^r \alpha_{ext}(r',\lambda)\,dr'\right]$$

Full lidar equation

Points to remember #4

- Putting all these terms together we find

$$P(r,\lambda) = P_0 \frac{c\tau}{2} A\eta(\lambda) \frac{O(r)}{r^2} \beta(r,\lambda) \exp\left[-2\int_0^r \alpha_{ext}(r',\lambda)\,dr'\right]$$

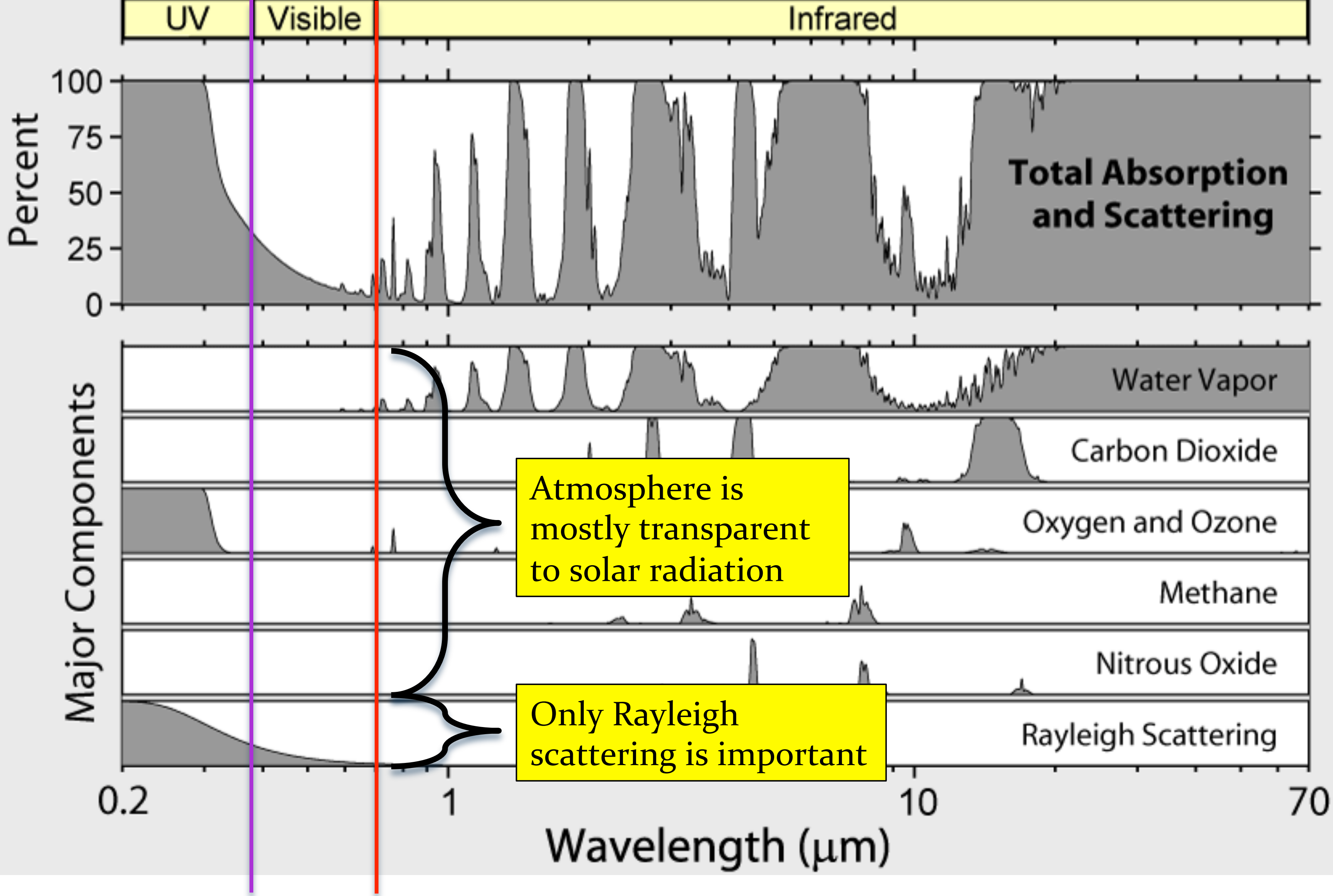
- And we need to remember that

$$\beta = \beta_{mol} + \beta_{par}$$

$$\alpha_{ext} = \alpha_{mol,ext} + \alpha_{par,ext}$$

Rayleigh scattering

Still 1 equation and 2 unknowns! Impossible to solve unless imposing other constrains.



$$\alpha_{mol,ext} = \alpha_{mol,scat} + \alpha_{mol,abs} \approx \alpha_{mol,scat}$$

Molecular cross-section

Total cross-section for Rayleigh scattering from molecules in the atmosphere is given by Bucholtz (1995):

$$\sigma_{mol}(\lambda) = \frac{24\pi^3}{\lambda^4 N_{std}^2} \frac{(n_{std}^2 - 1)^2}{(n_{std}^2 + 2)^2} \frac{6 + 3\rho_n}{6 - 7\rho_n}$$

- $n_{std}(\lambda)$ is the index of refraction of dry air
- $\rho_n(\lambda)$ is the depolarization factor
- N_{std} is the standard molecular density

Standard Atmosphere, N_{std}

Again according to Bucholtz (1995)

- The standard atmosphere is defined as dry air with 0.03% de CO_2 by volume (300 ppmv) with pressure 1013.25 hPa and temperature 15°C.

- In the standard atmosphere
  - $N_{std} = 2.54743 \times 10^{19} \text{cm}^{-3}$

Index of refraction, n_{std}

- The $\mathbf{n_{std}}$ can be calculated using the equations given by Peck and Reeder (1972), λ is given in μm
- For λ < 230nm

$$(n_{std} - 1) \times 10^8 = \frac{5791817}{238.0185 - \lambda^{-2}} + \frac{167909}{57.362 - \lambda^{-2}}$$

- For λ > 230nm

$$(n_{std} - 1) \times 10^8 = 8060.51 + \frac{2480990}{132.274 - \lambda^{-2}} + \frac{17455.7}{39.32957 - \lambda^{-2}}$$

355nm	n-1 = 2.86×10^{-4}
532nm	n-1 = 2.78×10^{-4}
1064nm	n-1 = 2.74×10^{-4}

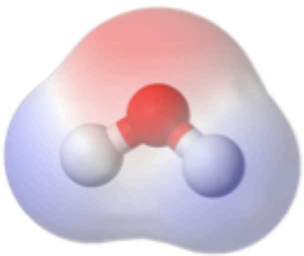
Anisotropy factor, F_k

- Rayleigh scattering by molecules is not completely isotropic.

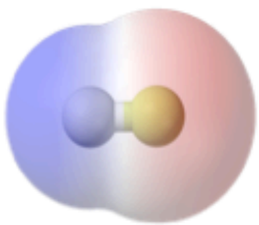
- The King correction factor, F_k, accounts for this anisotropy

$$F_k = \frac{6 + 3\rho_n}{6 - 7\rho_n}$$

...NOT ALL MOLECULES ARE EQUAL TO SCATTERING

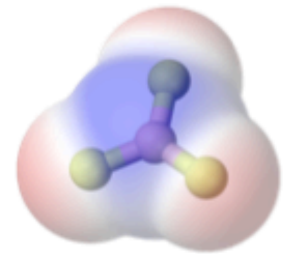


H_2O

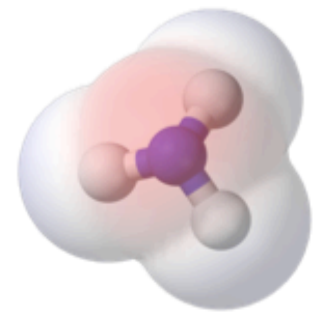


HF

DISPLAY AN INTERNAL CHARGE GRADIENT (PERMANENT ELETRIC FIELD)



NH_3



CH_4

SUM OF ALL CHARGES & FIELD EQUALS TO ZERO (SIMMETRIES)

Depolarization factor

- The depolarization factor depends on the molecule and on the wavelength. Bates (1984) gives the following equations for λ in μm

$$
\begin{aligned}
F(N_2) &= 1.034 + 3.17 \times 10^{-4} \frac{1}{\lambda^2} \\
F(O_2) &= 1.096 + 1.385 \times 10^{-3} \frac{1}{\lambda^2} + 1.448 \times 10^{-4} \frac{1}{\lambda^4} \\
F\ (\mathrm{Ar}) &= 1.00 \\
F(CO_2) &= 1.15
\end{aligned}
$$

Depolarization factor of air

- According to Bodhaine et al (1999), these can be combined to obtain the depolarization factor of dry air considering the mass fractions of each gaseous species

$$F(air) = \frac{\sum_k C_k F_k}{\sum_k C_k}$$

355nm	$F_k^{air} = 1.053$	$\rho_n^{air} = 0.0306$
532nm	$F_k^{air} = 1.049$	$\rho_n^{air} = 0.0284$
1064nm	$F_k^{air} = 1.047$	$\rho_n^{air} = 0.0274$

$$F(air, CO_2) = \frac{78.084F(N_2) + 20.946F(O_2) + 0.934 \times 1.00 + C_{CO_2} \times 1.15}{78.084 + 20.946 + 0.934 + C_{CO2}}$$

Molecular Scattering coefficient

- With the molecular scattering cross-section, we can calculate the molecular scattering coefficient

$$\alpha_{scat,mol}^{std}(\lambda) = N^{std}\sigma_{scat,mol}(\lambda)$$

355nm	$\alpha_{std} = 70.3\ Mm^{-1}$
532nm	$\alpha_{std} = 13.2\ Mm^{-1}$
1064nm	$\alpha_{std} = 0.80\ Mm^{-1}$

- For different P and T conditions, what changes is **N**

$$\alpha_{mol}(\lambda,z) = \alpha_{mol}^{std}(\lambda)\frac{N(z)}{N^{std}} = \alpha_{mol}^{std}(\lambda)\frac{P(z)}{P^{std}}\frac{T^{std}}{T(z)}$$

Mol. Angular-scattering coefficient

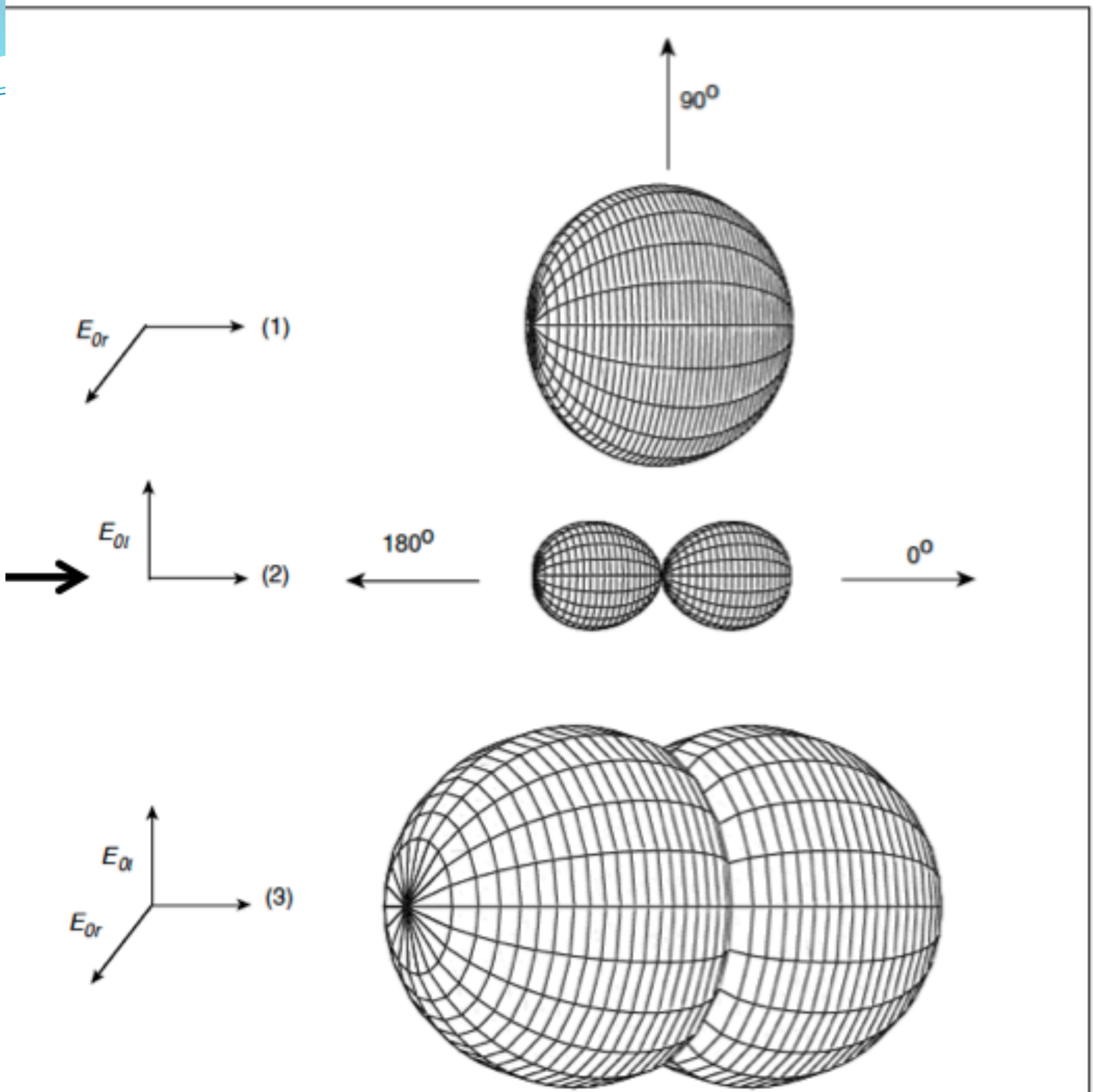
- From the previous discussion we know that:

$$\beta_{mol}(\theta, \lambda, z) = \frac{\alpha_{mol}(\lambda, z)}{4\pi} P_{mol}(\theta, \lambda)$$

- Where the molecular phase function is for Rayleigh scattering is

$$\boldsymbol{P}_{\mathrm{ray}}(\theta) = \tfrac{3}{4}(1 + \cos^2 \theta),$$

- This, however, does not include molecular anisotropy.


90°
E_{0r}
(1)
E_{0l}
180°
(2)
0°
E_{0l}
(3)
E_{0r}

Molecular phase function

- With anisotropy, according to Bucholtz (1995):

$$P_{mol}(\theta) = \frac{3}{4(1+2\gamma)}\left[(1+3\gamma)+(1-\gamma)cos^2\theta\right]$$

- The parameter γ comes from the the depolarization factor, ρ_n, we have shown before :

$$\gamma = \frac{\rho_n}{2-\rho_n}$$

355nm	$\gamma^{air} = 0.0155$
532nm	$\gamma^{air} = 0.0144$
1064nm	$\gamma^{air} = 0.0139$

Molecular backscatter

- Putting pieces together and taking θ=π, we find that

$$\beta_{mol}(\pi, \lambda, z) = \frac{\alpha_{mol}(\lambda, z)}{8\pi/3} \frac{1+\gamma}{1+2\gamma}$$

- Which means that the molecular lidar ratio is

$$LR_{mol} = \frac{\alpha_{mol}}{\beta_{mol}} = \frac{8\pi}{3}\frac{1+2\gamma}{1+\gamma} = \frac{8\pi}{3}\frac{2+\rho_n}{2}$$

355nm	$LR_{mol} = 1.0153$	$\times \frac{8\pi}{3}$
532nm	$LR_{mol} = 1.0142$	$\times \frac{8\pi}{3}$
1064nm	$LR_{mol} = 1.0137$	$\times \frac{8\pi}{3}$

Last term corrected on 16th Aug'15

Molecular signal

- The optical depth due to molecular extinction long the range **r** is

$$\tau_{mol}(\lambda,r)=\int_0^r \alpha(\lambda,r')\,dr'=\alpha_{mol,std}\frac{T_{std}}{P_{std}}\int_0^r \frac{P(r')}{T(r')}\,dr'$$

- Hence the molecular lidar signal is written as

$$P_{mol}(r)\propto\frac{1}{r^2}\beta_{mol}(r)\exp\left[-2\alpha_{mol}^{std}\frac{T^{std}}{P^{std}}\int_0^r \frac{P(r')}{T(r')}\,dr'\right]$$

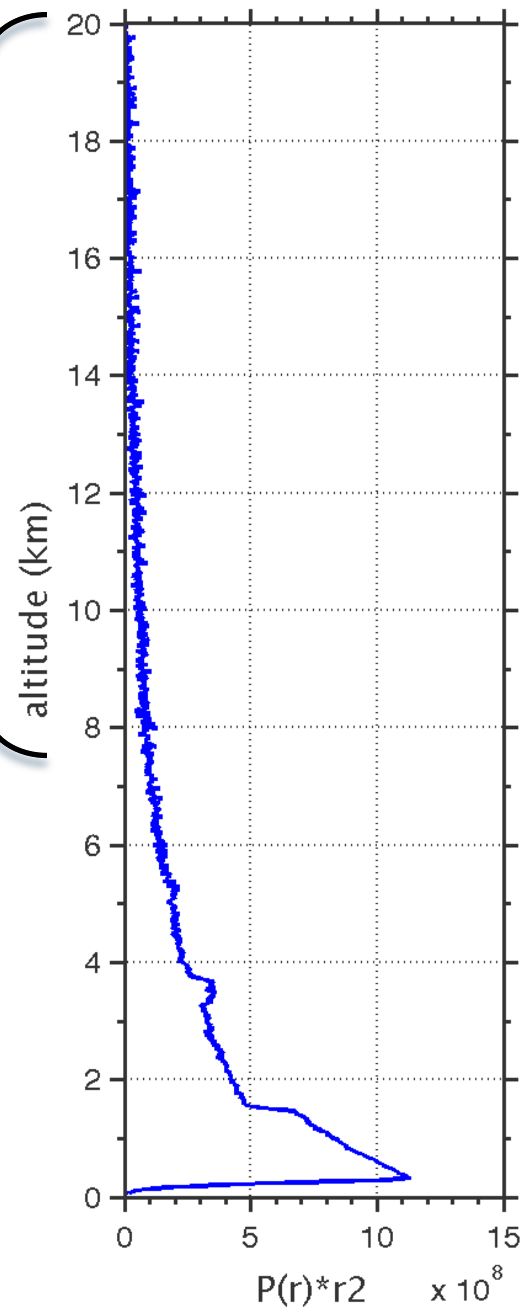
What is the proportionality constant?

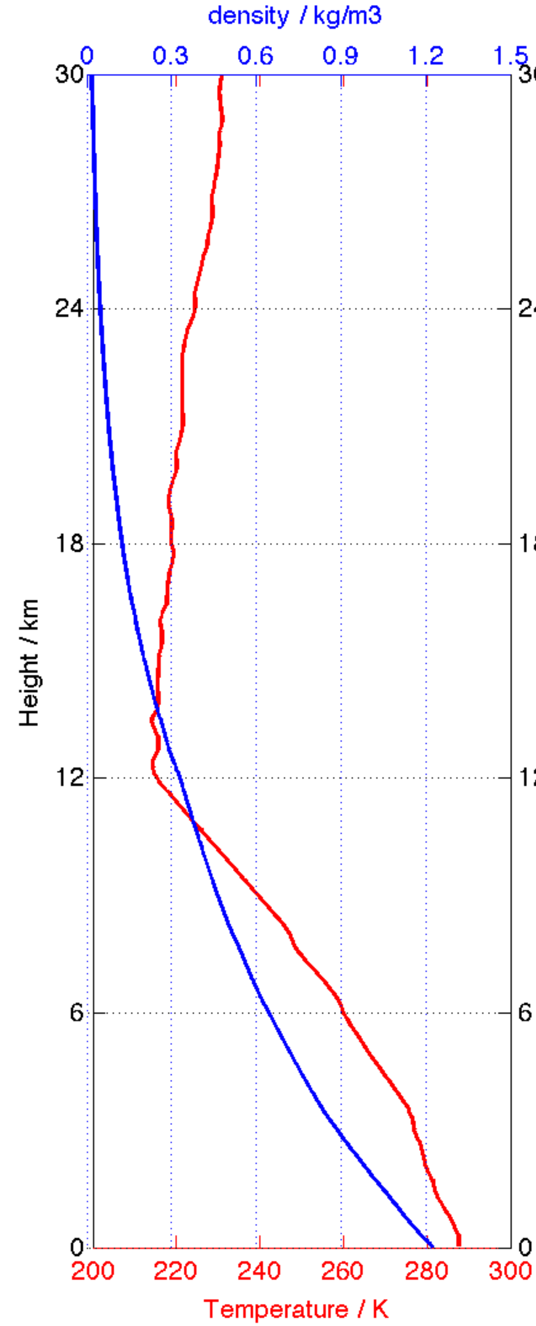
Molecular fit

This part of the signal looks like just molecular

- Fit:

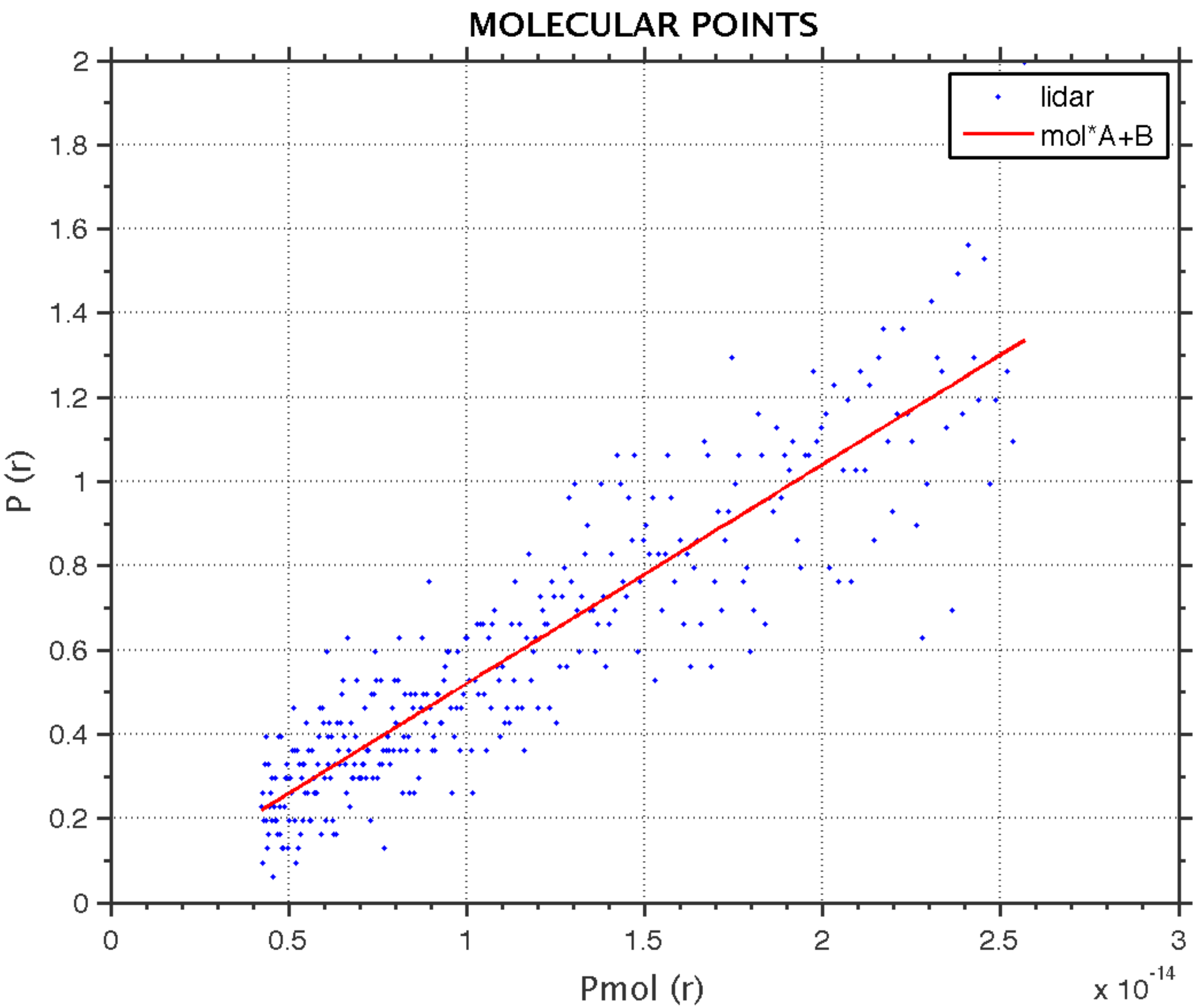
$$P(r)=A*P_{mol}(r)+BG$$

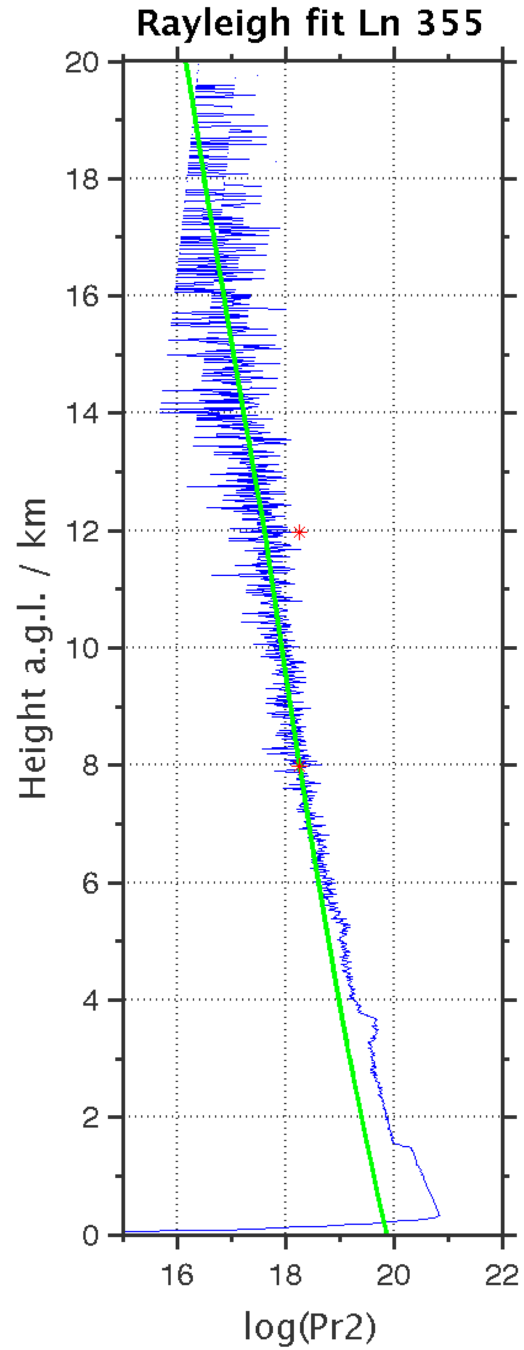




Earlinet, paper #1

Molecular fit





Solutions to the Lidar equation

- Rewrite the equation as:

$$r^2 P(r,\lambda) = C\beta(r,\lambda)\exp\left[-2\int_0^r \alpha_{ext}(r',\lambda)\,dr'\right]$$

- And consider a new variable:

$$S(r) = \log\left(r^2 P(r,\lambda)\right)$$

- Then

$$S(r) = \log(C) + \log(\beta(r)) - 2\int_0^r \alpha(r',\lambda)\,dr'$$

If homogeneous atmosphere

- Taking the derivative to **r**

$$\frac{dS}{dr} = \frac{1}{\beta}\frac{d\beta}{dr} - 2\alpha$$

- If the atmosphere is homogenous, **β = cte**, then

$$\alpha = -\frac{1}{2}\frac{dS}{dr}$$

- We just need a linear-fit where S(r) is a straight line. This is the **slope method**.

Analytical methods

Hitschfeld & Bordan, J. Meteo. 1954 radar

- Fernald, Ap. Opt. 1972 Forward $\beta = B\alpha$
- Klett, Ap. Opt. 1981 Backward $\beta = B\alpha^K$
  - Turbid, $\alpha_p >> \alpha_m \sim 0$
- Fernald, Ap. Opt. 1984 $\beta_m = B_m\alpha_m$ $\beta_p = B_p\alpha_p$
  - Turbid, $\alpha_p \sim \alpha_m > 0$
- Klett, Ap. Opt. 1985 $\beta = B(r)\alpha^K$
- Sasano et al, Ap. Opt. 1985 $\beta_m = B_m\alpha_m$ $\beta_p = B_p(r)\alpha_p$

Other methods

Slope method

- Collis, QJRMS 1966
- Viezee et al, JAM 1969

Inverse modeling

- Kastner, 1987
- Yee, 1989

Total Integrated backscatter

Klett-Fernald-Sazano

- Following Ansmann & Muller (Weitkamp, chap 4):

$$\beta_{aer}(R) + \beta_{mol}(R) = \frac{S(R)\exp\left\{-2\int_{R_0}^{R}[L_{aer}(r) - L_{mol}]\,\beta_{mol}(r)\,\mathrm{d}r\right\}}{\frac{S(R_0)}{\beta_{aer}(R_0) + \beta_{mol}(R_0)} - 2\int_{R_0}^{R} L_{aer}(r)S(r)T(r, R_0)\,\mathrm{d}r}$$

- Where:

$$T(r, R_0) = \exp\left\{-2\int_{R_0}^{r}[L_{aer}(r') - L_{mol}]\,\beta_{mol}(r')\,\mathrm{d}r'\right\}$$

$$L_{aer}(R) = \frac{\alpha_{aer}(R)}{\beta_{aer}(R)}$$

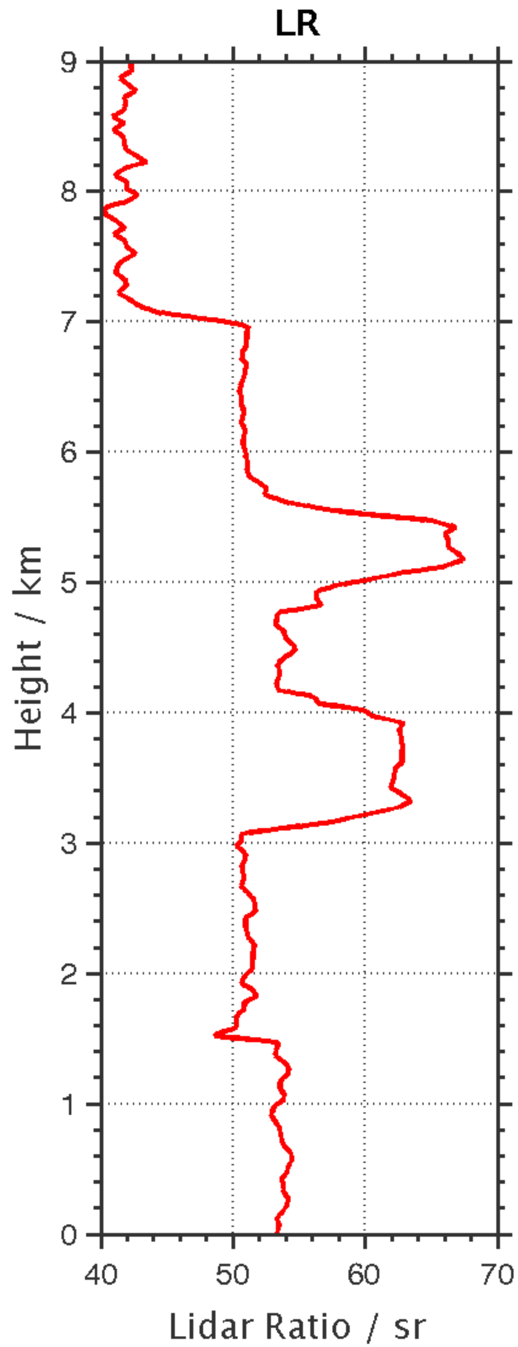
$$L_{mol} = \frac{\alpha_{mol}(R)}{\beta_{mol}(R)}$$

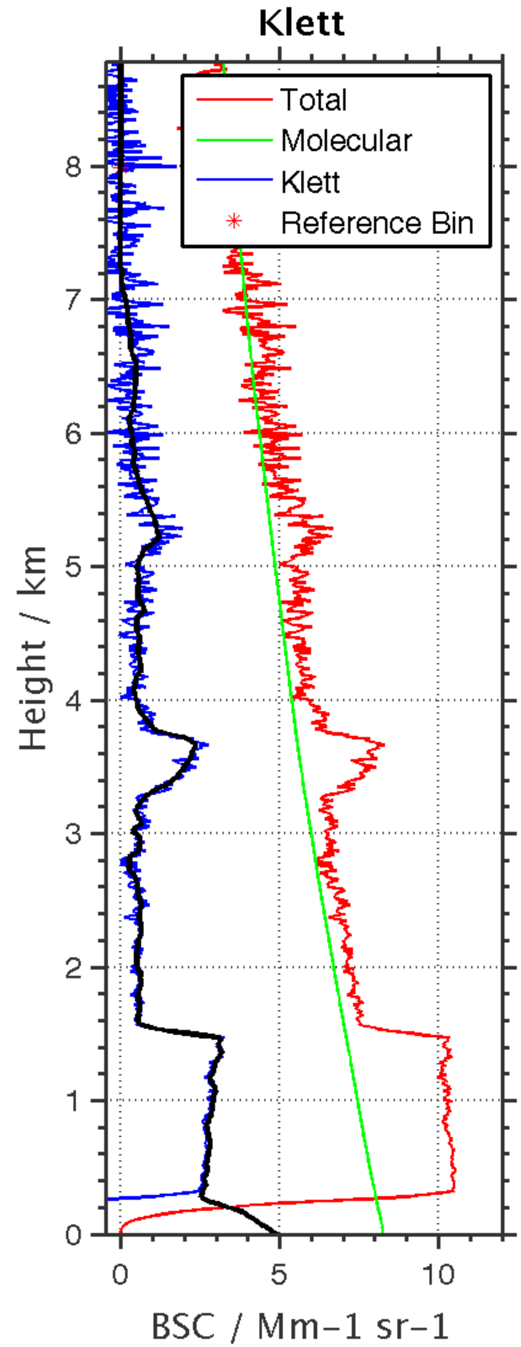
Klett-Fernald-Sazano

- Ansmann & Muller (Weitkamp, chap 4):
  - "… can, in principle, be integrated by starting from the reference range R_0, which may be either the near end ($R > R_0$, forward integration) or the remote end ($R < R_0$, backward integration) of the measuring range. Numerical stability, which is not to be mistaken for accuracy, is, however, given only in the backward integration case".

But what if we used 40 sr or 70 sr?

How much would the solution change?





Typical LR_{aer}

- Ansmann (WLMLA Course, 2013 Pucon)
  - "Forward and Backward Klett should converge when using the correct particle lidar ratio"

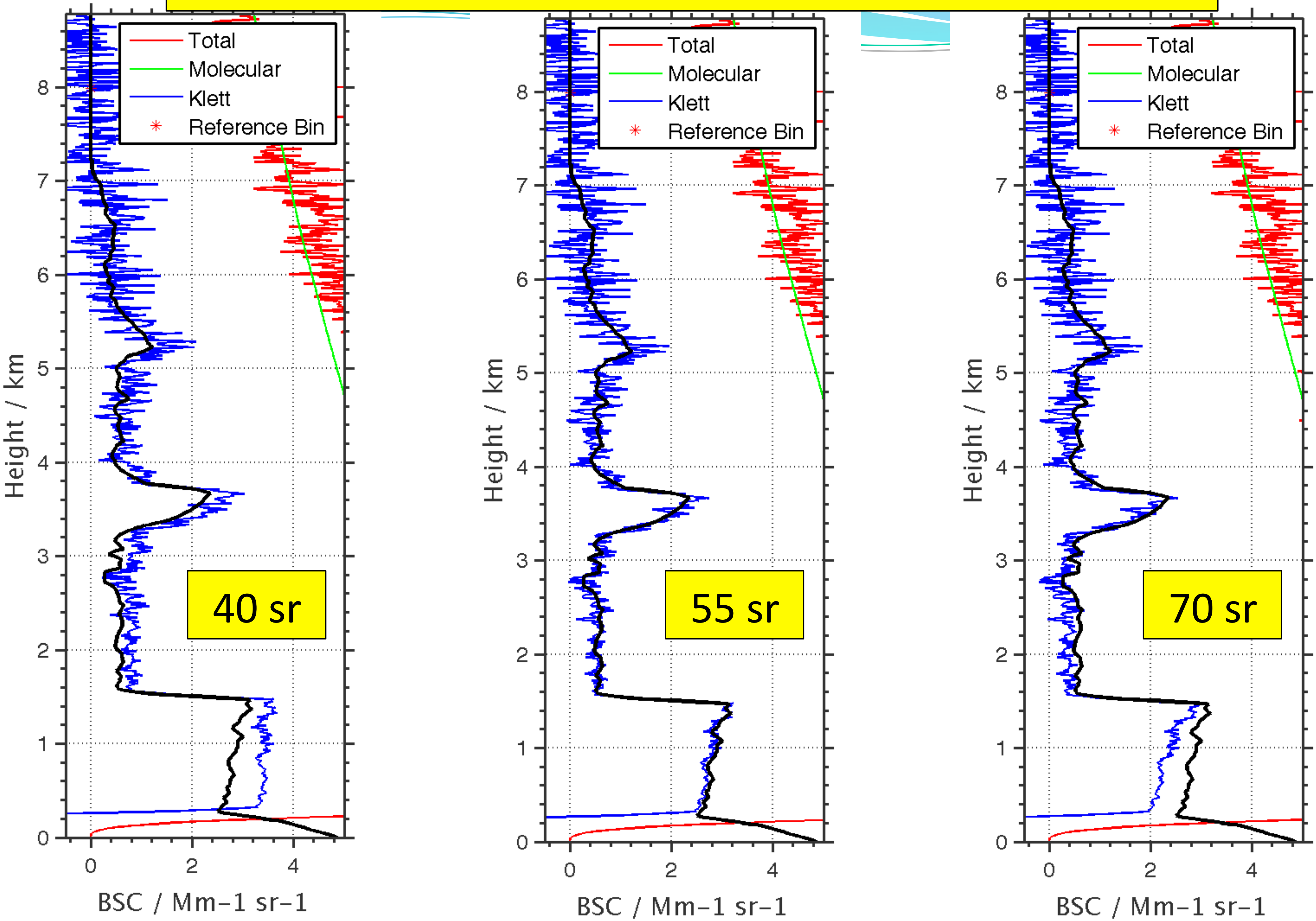
WLMLA-Course 2011, Bolivia

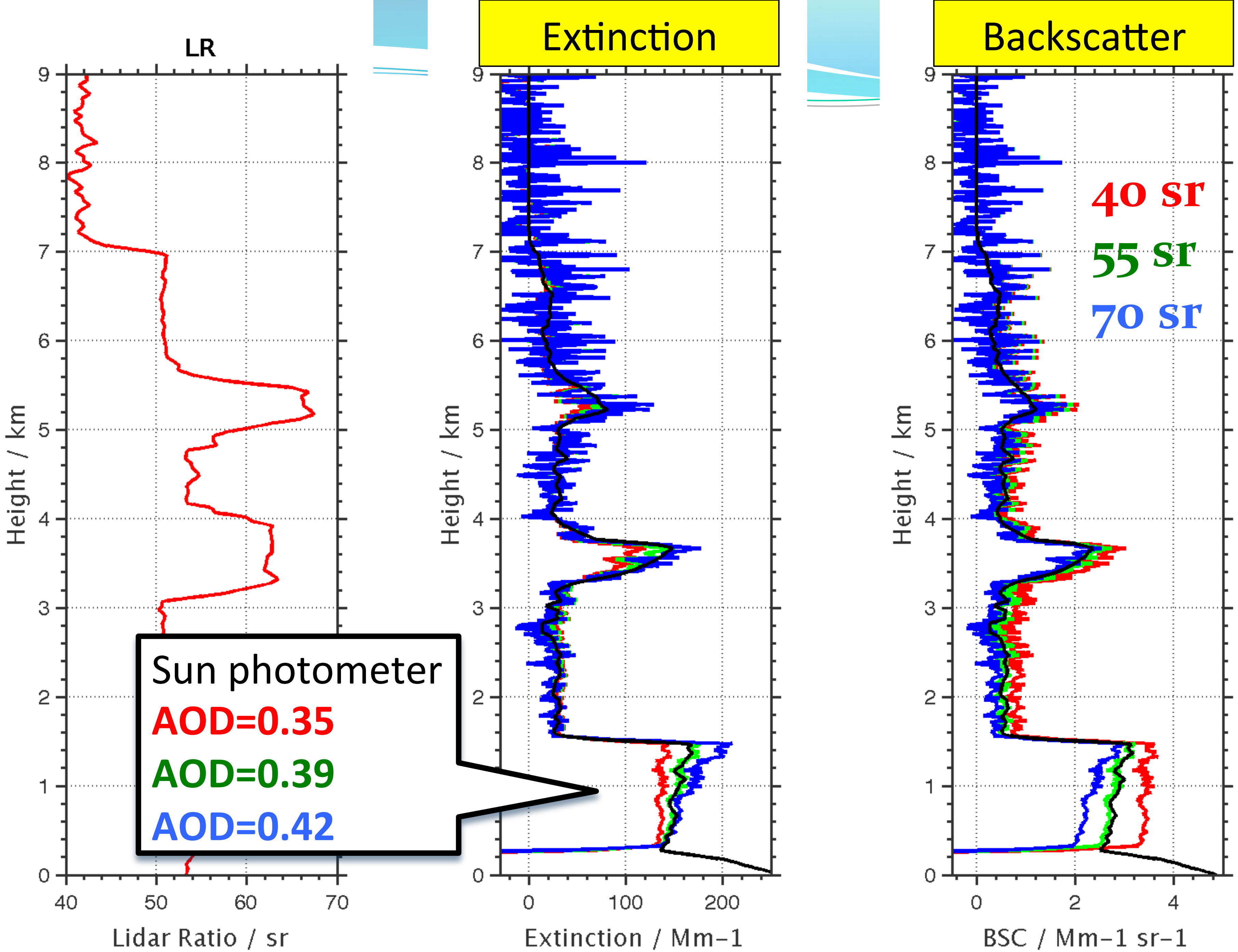
- Ice crystals 10 sr
- Marine 20-30 sr
- Continental 60-70 sr
- Heavy polluted 100 sr

WLMLA-Course 2013, Chile

- Marine 20-35 sr
- Saharan 40-70 sr
- Biomass 70-100 sr
- Urban 45-75 sr

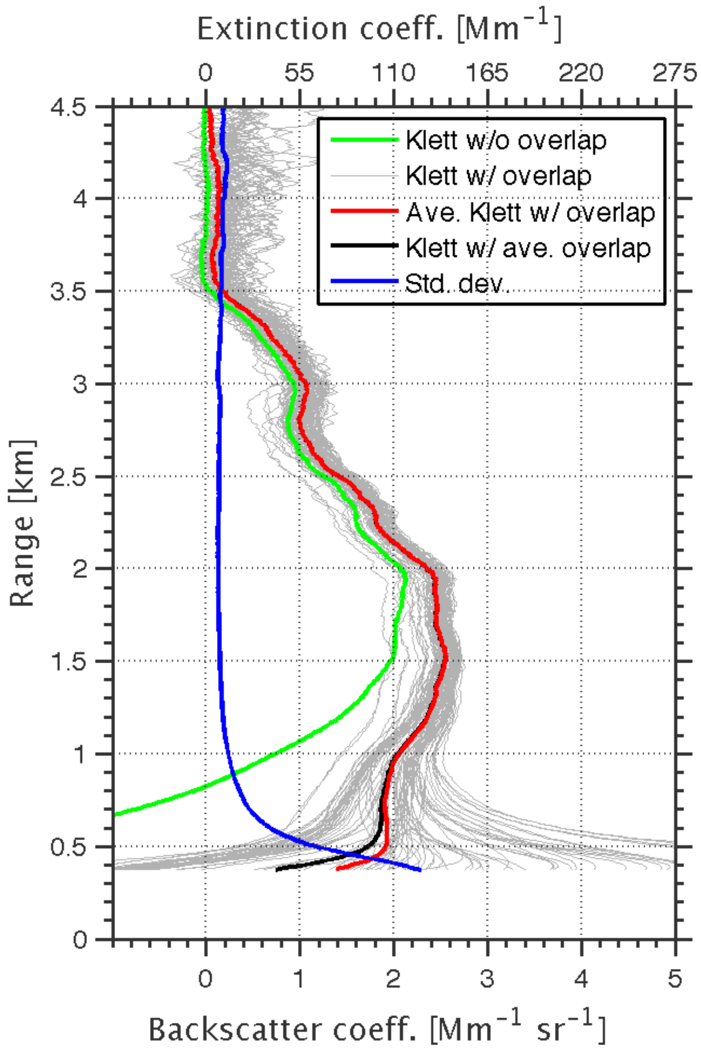
Backscatter from Klett-Fernald-Sasano

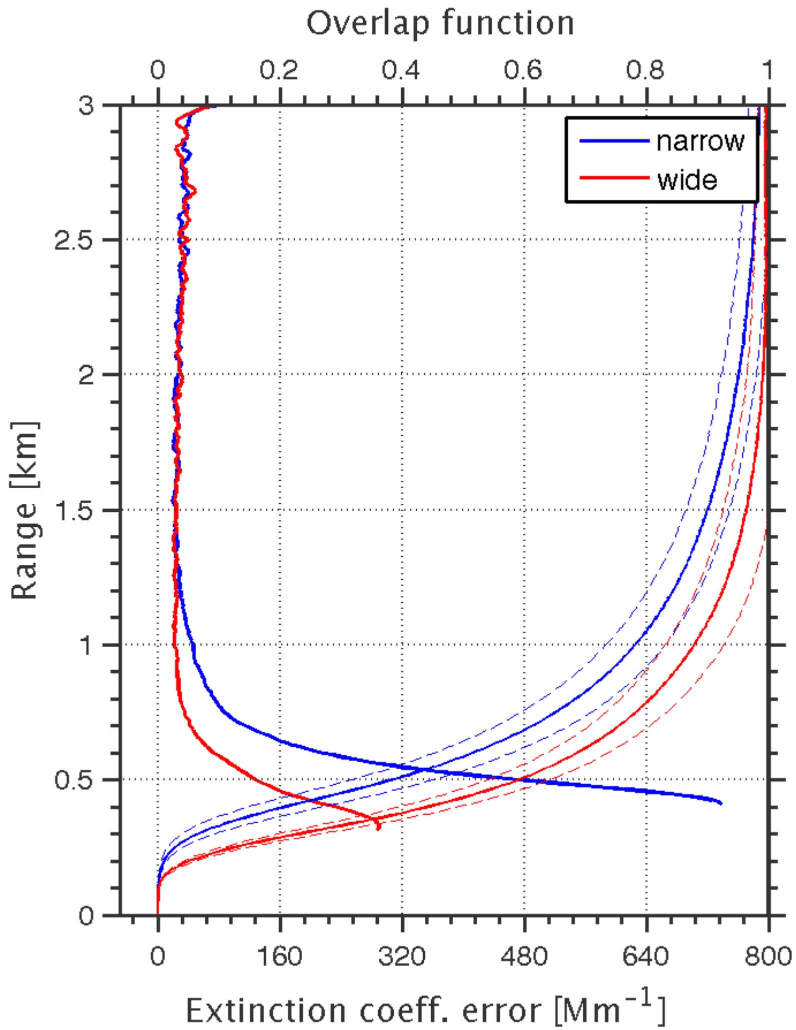




LR
Lidar Ratio / sr
Height / km
Extinction
Extinction / Mm-1
Backscatter
BSC / Mm-1 sr-1
40 sr
55 sr
70 sr
Sun photometer
AOD=0.35
AOD=0.39
AOD=0.42

Another source of error, Overlap





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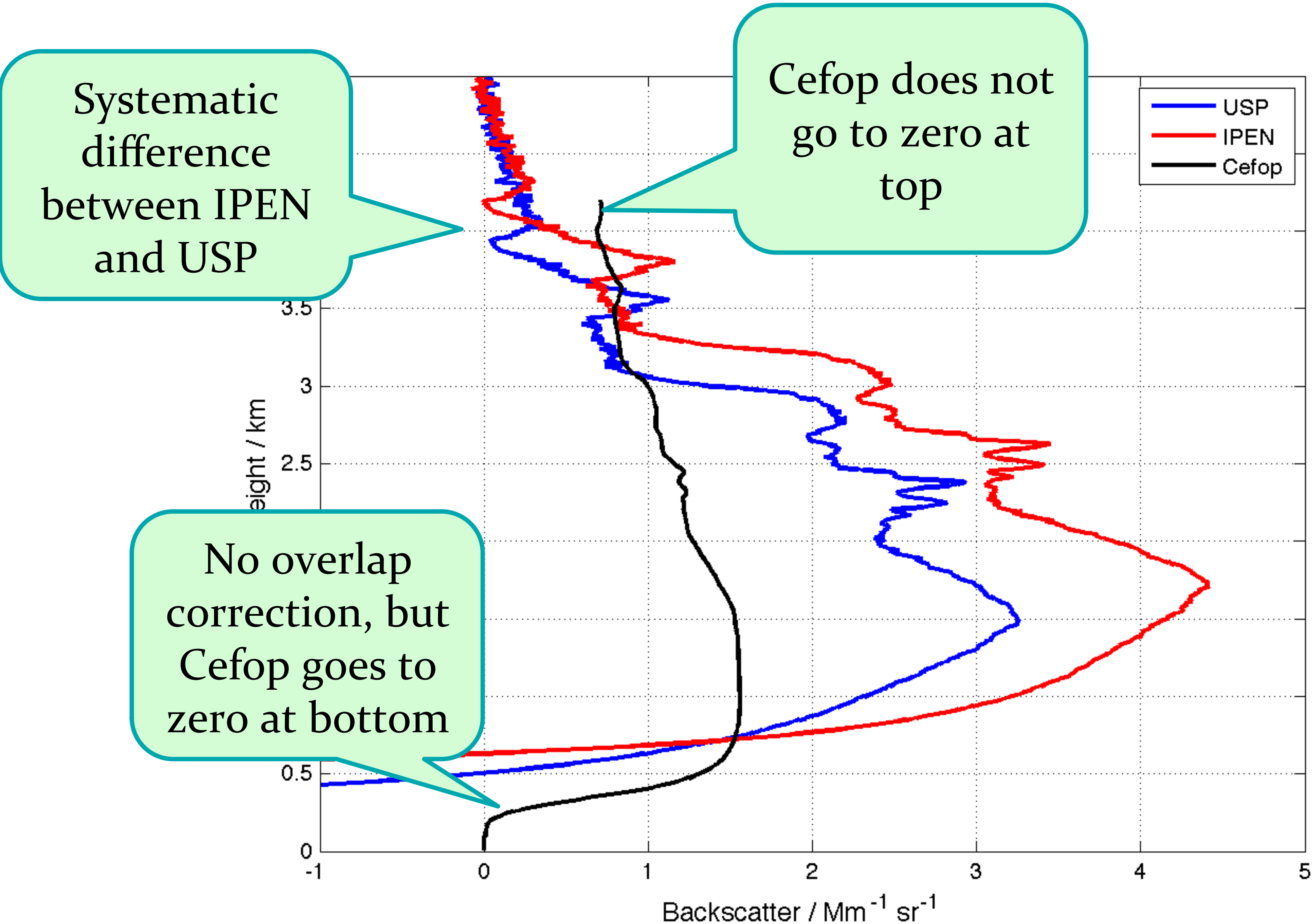
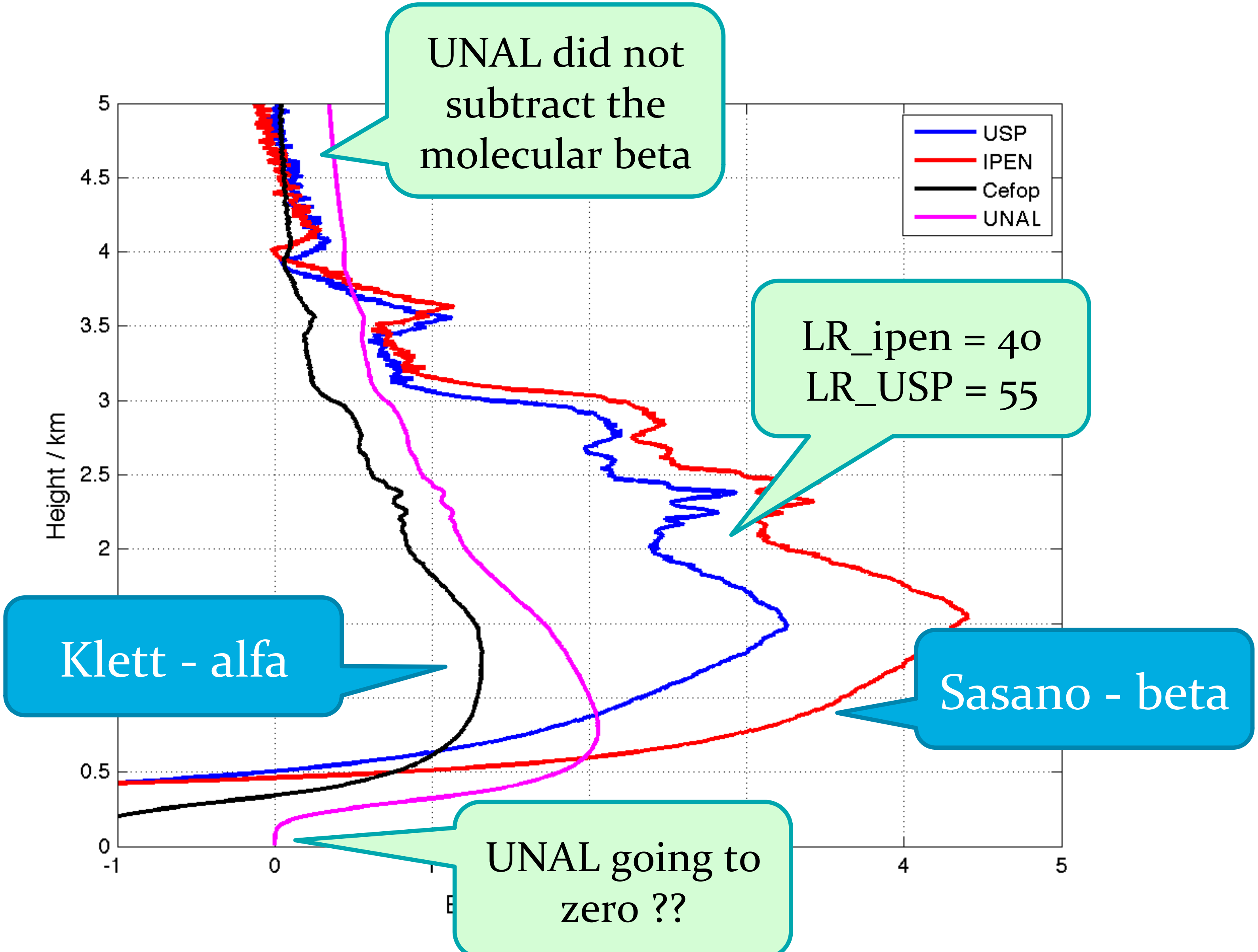
  - Molecular atmosphere
  - Klett-Fernald-Sasano
- **The Lalinet (tentative) algorithm**

Round #1, Pucon 2013



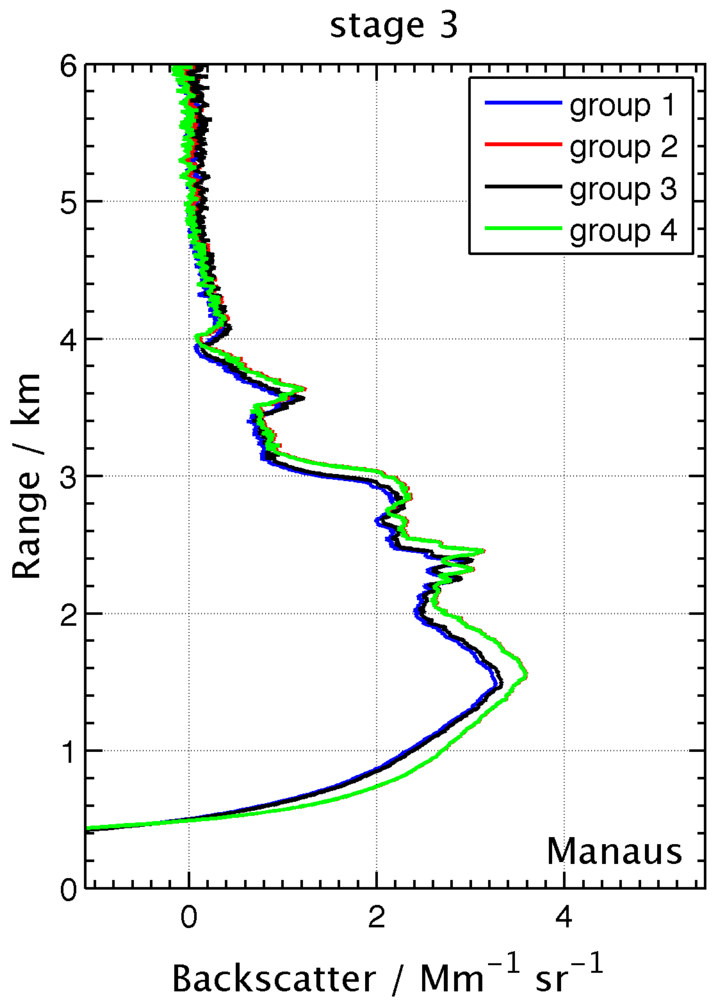
Round #2


UNAL did not subtract the molecular beta
LR_ipen = 40
LR_USP = 55
Klett - alfa
Sasano - beta
UNAL going to zero ??
USP
IPEN
Cefop
UNAL
Height / km
5
4.5
4
3.5
3
2.5
2
0.5
0
-1
0
4
5

Round #3 for Barbosa et al, OPA 2014

- After 3 to 5 stages of intercomparison, searching for bugs, fixing, etc… we got to a point where we could trust our analysis.

- But still, systematic differences!



Workshop, Concepcion 2014

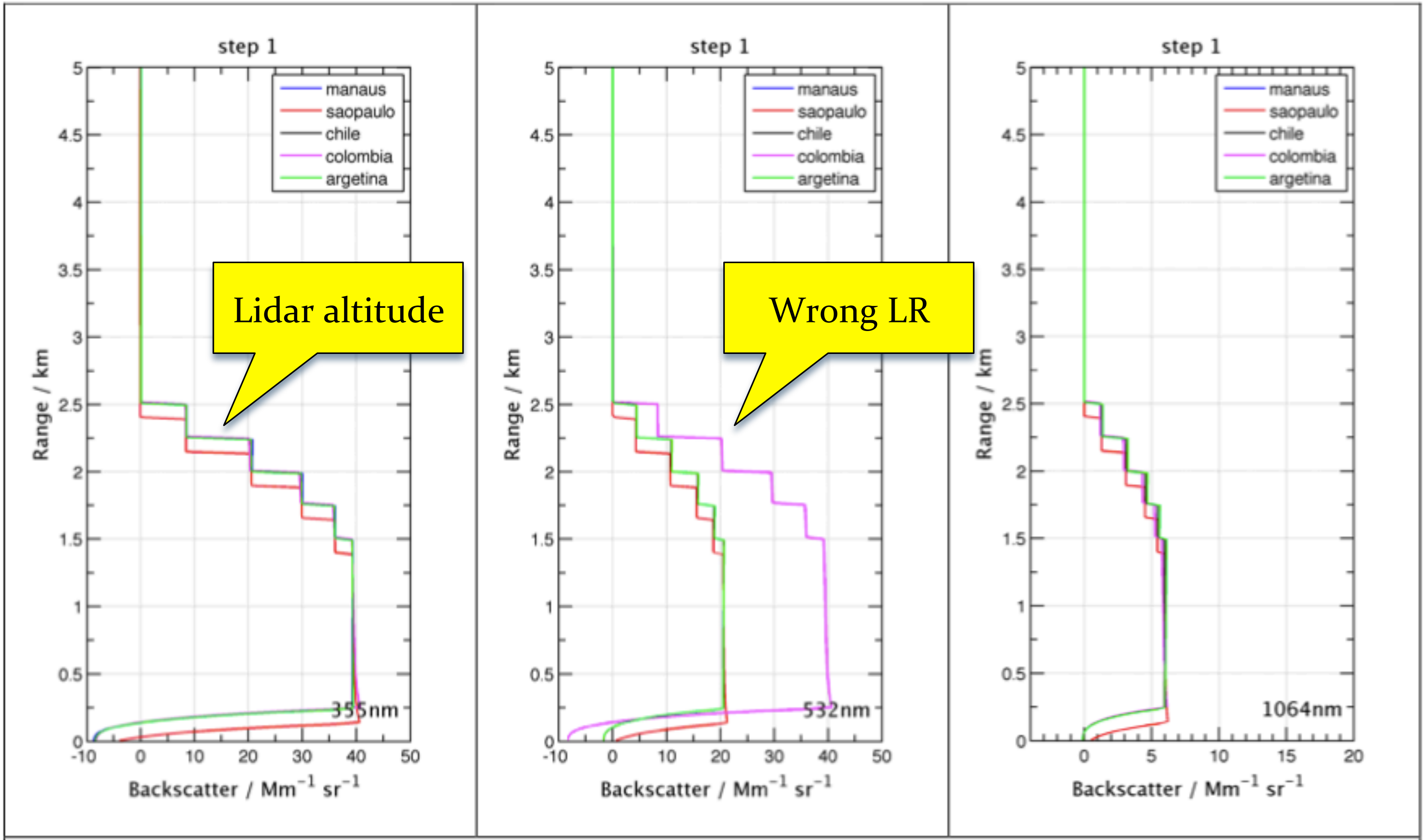


Figure 1 – Particle backscatter (Mm-1 sr-1) after step 1.

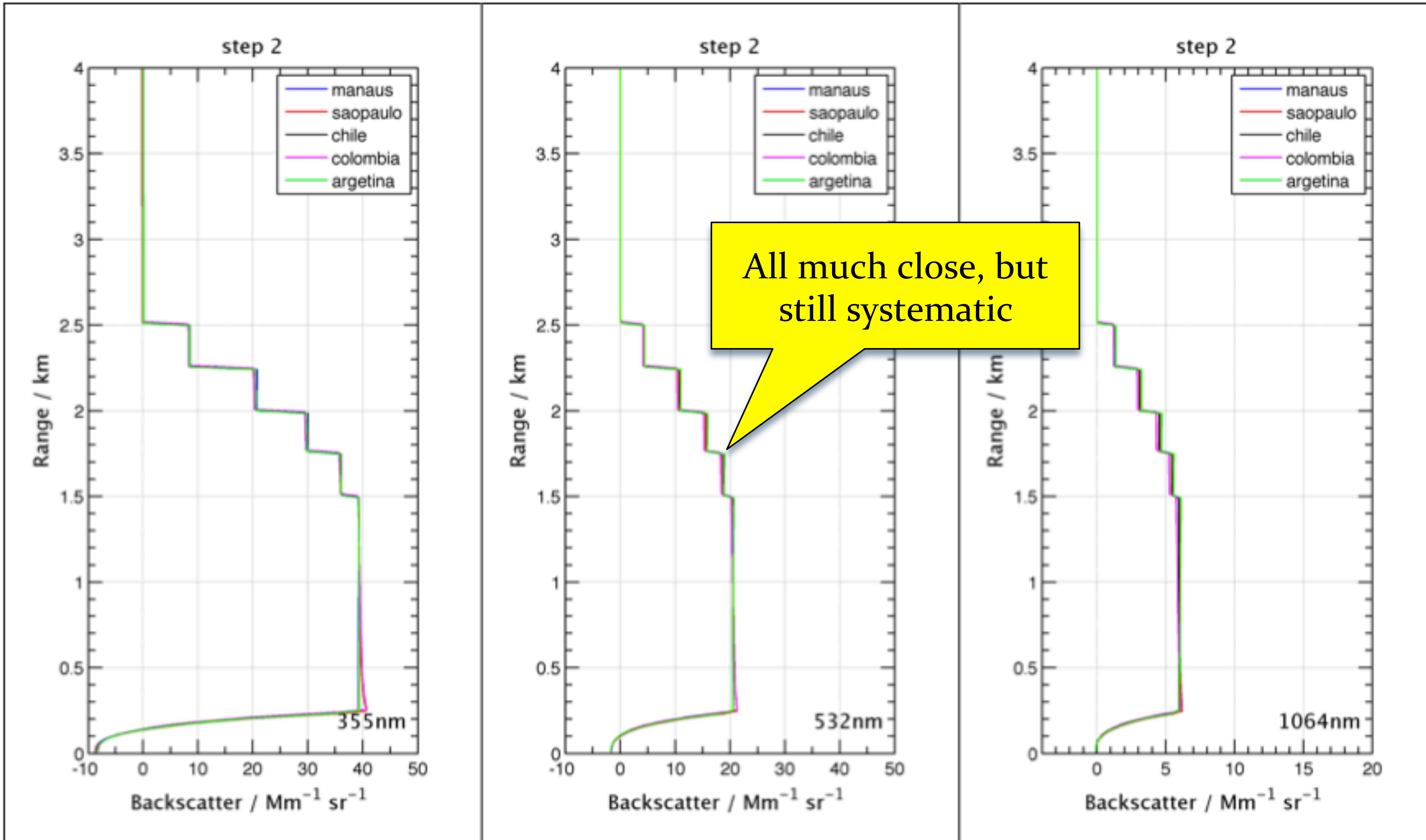


Figure 2 – Particle backscatter (Mm-1 sr-1) after step 2.

Table 1 – Differences before the first step (black), after the comparison of molecular quantities and first changes to the inversion algorithms (red) and after the comparison with known answer (green) are shown.

	$\beta_{par}(z_0)$ Mm^{-1}	$\beta_{mol}(z_0)$ Mm^{-1}	**Scale** $x10^{14}$	**BG**	**Bin** z_0	**σstd** 10^{-30} m^{-2}	**LR** sr^{-1}
AM	0	1.6157 1.6054	4.8967 4.89763	$1.2294\ 10^{-3}$ $7.6318\ 10^{-4}$	899 900	2.7694	8.5058
SP	0	1.60404	1.748 1.75087 4.8952	- $8.16868\ 10^{-7}$ $6.045\ 10^{-4}$	900	2.7606	8.50411
CH	0	1.5598 1.6065	2.08 2.034 4.8859	0 $8.33\ 10^{-4}$	900	2.6381 2.7694	8.3776 8.5058
CO	0	1.51 1.60404	1.8489 1.75087 4.8929	- $8.16868\ 10^{-7}$ $5.5\ 10^{-4}$	900	2.7606	8.50411
AR	10^{-3}	1.5597 1.609	4.83 4.89	$4.81\ 10^{-12}$ $1.16\ 10^{-3}$ $7.34\ 10^{-4}$	900	2.639 2.7694	8.3776 8.5058

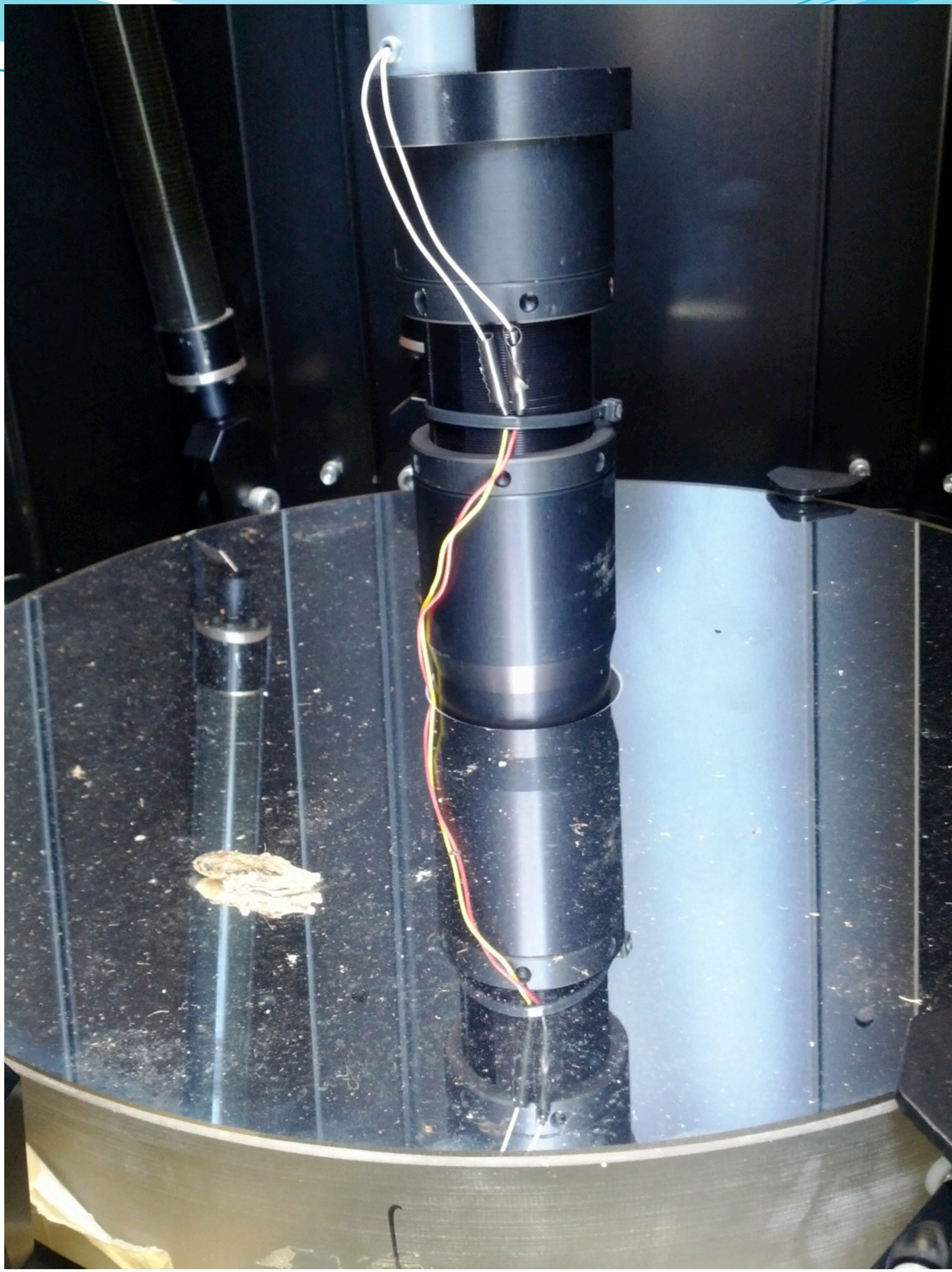
Manu bugs…

AM was missing a 2 in the attenuation term exp[-2τ] of the molecular signal

SP and CO were using βmol(z) instead of Pmol(z) for the fitting.

CH was removing the background from the signal but not from the range corrected signal.

AR was modifying the lidar signal instead of the synthetic molecular signal, which leads to fitting errors when the lidar signal is very noise.



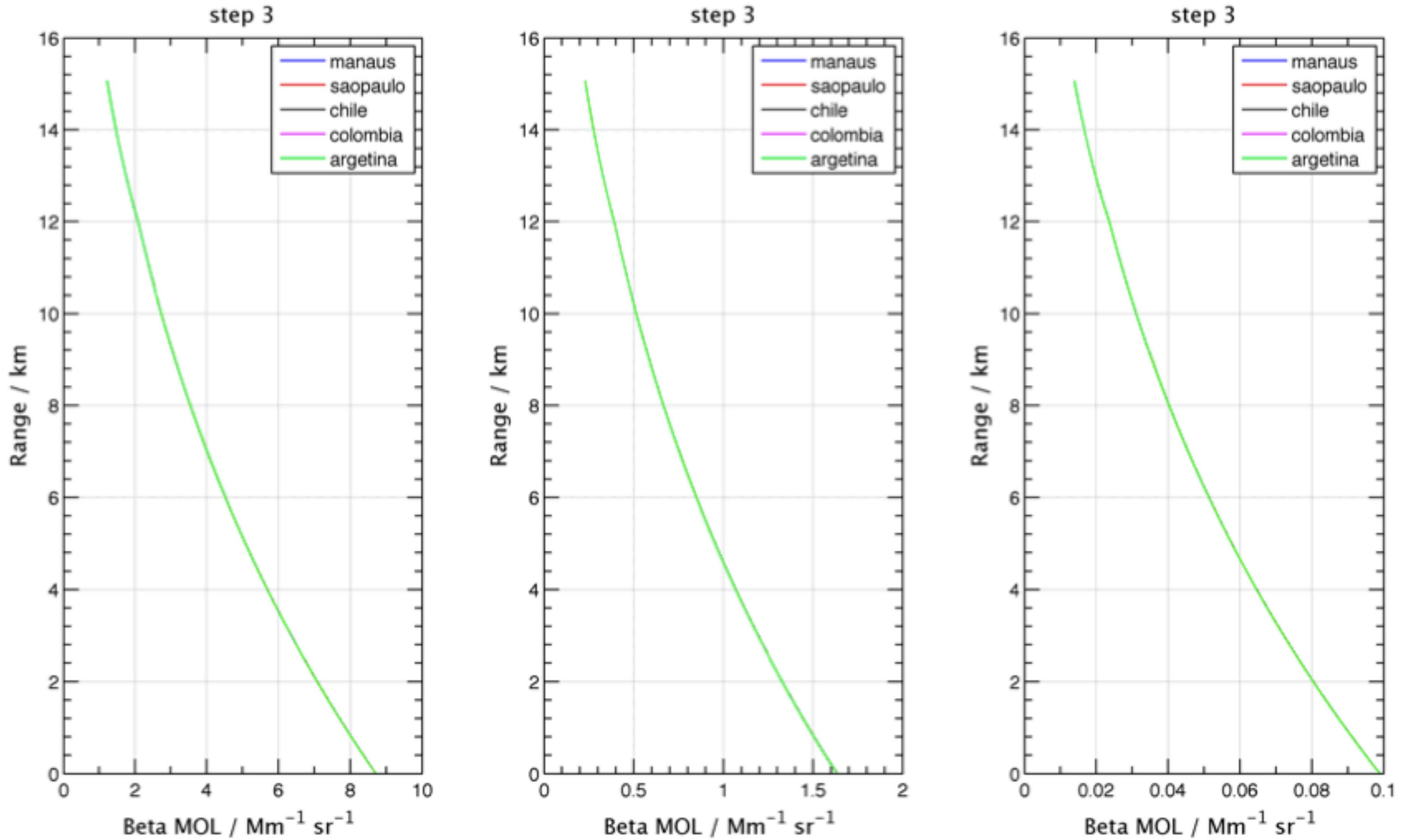
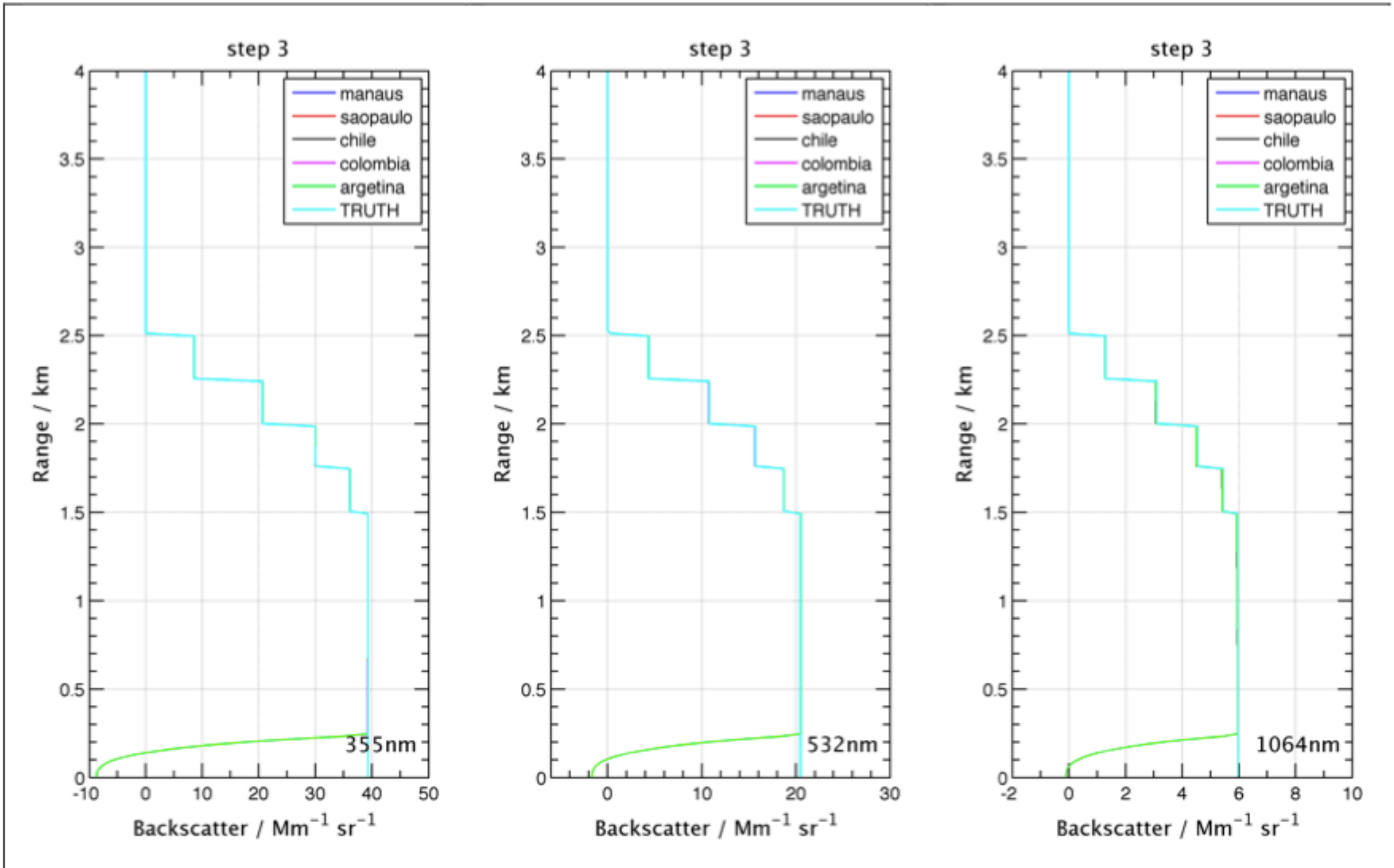


Figure 3 – Molecular backscatter (Mm-1 sr-1) at begin of step 3.


step 3
manaus
saopaulo
chile
colombia
argetina
TRUTH
Range / km
Backscatter / Mm^{-1} sr^{-1}
355nm
step 3
manaus
saopaulo
chile
colombia
argetina
TRUTH
Range / km
Backscatter / Mm^{-1} sr^{-1}
532nm
step 3
manaus
saopaulo
chile
colombia
argetina
TRUTH
Range / km
Backscatter / Mm^{-1} sr^{-1}
1064nm

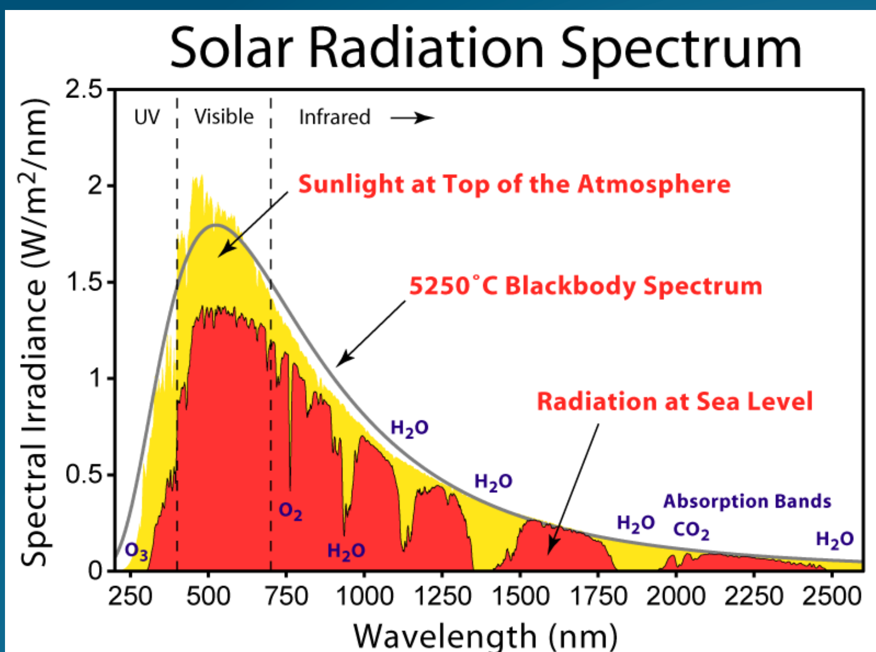
Lalinet code

Available code in Matlab and Mathematica

- Molecular atmosphere
- Rayleigh fit with BG
- Klett

Next steps:

- Auto-find molecular region
- Auto-find clouds
- Overlap
- Raman
- etc…



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