Applied Optics

Letters to the Editor

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Lidar calibration and extinction coefficients

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Received 12 July 1982.

In a recent article in this journal,[1] a lidar inversion scheme was presented which utilizes only relative changes in the backscattered signal to produce estimates of atmospheric extinction as a function of range. It was also pointed out that better results could be obtained if one could make use of calibration information and/or other independent measurements to determine reference or boundary values of extinction σ_m more accurately. This Letter describes some characteristic features of a particular boundary value model for estimating σ_m based on knowledge of (1) lidar system constants, (2) relative signal strength, and (3) the relationship between the backscatter and extinction coefficients.

If one assumes that the atmospheric backscatter and extinction coefficients, β and σ, respectively, are related according to a power law of the form

$$\beta = B\sigma^k, \tag{1}$$

where B and k depend on wavelength and various properties of the obscuring aerosol, the single-scattering lidar equation may be expressed in the form

$$S(r) \equiv \ln[r^2P(r)] = C_1 + k\ln\sigma - 2\int_0^r \sigma dr'. \tag{2}$$

In this equation $P(r)$ is the instantaneous power received from range r, and the constant C_1 is given by $C_1 = \ln(0.5P_0Bc\tau A)$, where P_0 is the transmitted power, c is the velocity of light, τ is the pulse duration, and A is the effective system receiver area.

For constant k a stable solution to Eq. (2) is

$$\sigma(r) = \frac{\exp[(S - S_m)/k]}{\left\{\sigma_m^{-1} + \dfrac{2}{k}\displaystyle\int_r^{r_m} \exp[(S - S_m)/k]dr'\right\}}, \tag{3}$$

where $S_m = S(r_m)$, $\sigma_m = \sigma(r_m)$, and $r \le r_m$.[1] Note that to apply this solution one must know σ_m as well as the relative signal $S - S_m$. For optical depths greater than unity, an adequate estimate for σ_m can usually be obtained from the well-known slope method[2] as minus one half of the average slope of the $S(r)$ curve. However, this approach becomes often quite inaccurate for less turbid atmospheres.

An obvious plausible strategy for obtaining an improved estimate of σ_m is to make use of the magnitude of the return signal S in addition to its relative strength $S - S_m$ over the range of interest. However, a characteristic difficulty with Eq. (2) arises in this connection, namely, that it can be applied only for $r \ge r_0$, where r_0 is the point of transmitter and receiver beam overlap. Thus in effect the receiver area A becomes an unknown function of r for $r < r_0$, and consequently it is impossible to solve for σ over $(0,r_0)$. In view of this limitation, an appropriate modification of the integral term in Eq. (2) is in order: Impose the additional constraint that the average extinction over $(0,r_m)$ is the same as that over (r_0,r_m); i.e., set

$$\bar{\sigma}_m \equiv r^{-1}\int_0^{r_m} \sigma dr' = (r_m - r_0)^{-1}\int_{r_0}^{r_m} \sigma dr' \equiv \langle \sigma_m \rangle, \tag{4}$$

so that for $r = r_m$ Eq. (2) becomes

$$S_m = C_1 + k\ln\sigma_m - 2r_m\langle\sigma_m\rangle. \tag{5}$$

The replacement of $\bar{\sigma}_m$ by $\langle\sigma_m\rangle$ will introduce a negligible error even for many strongly heterogeneous distributions of σ, so long as the overlap point r_0 is small compared with the maximum useful range r_m.

Now by integrating Eq. (3) over (r_0,r_m) one can obtain an expression for $\langle\sigma_m\rangle$ in terms of σ_m; on substituting the result into Eq. (5) an equation relating σ_m to known quantities is finally obtained. In dimensionless form the relationship is

$$G_m = \ln\Omega_m - \frac{r_m}{(r_m - r_0)}\ln(1 + I\Omega_m), \tag{6}$$

where $\Omega_m \equiv 2\sigma_m(r_m - r_0)/k$ (a measure of optical depth),

$$I \equiv (r_m - r_0)^{-1}\int_{r_0}^{r_m} \exp[(S - S_m)/k]dr',$$

$$G_m = \frac{(S_m - C_1)}{k} + \ln\left[\frac{2(r_m - r_0)}{k}\right] \tag{7}$$

$$= \ln\left\{\frac{2^{(k+1)/k}}{k}\left[\frac{P(r_m)}{P_0}\right]^{1/k}\left(\frac{r_m^2}{A}\right)^{1/k}\frac{(r_m - r_0)}{(c\tau B)^{1/k}}\right\}. \tag{8}$$

Since the quantity $(c\tau B)^{1/k}$ has the dimensions of a length (k is dimensionless), Eq. (8) shows explicitly that G_m is dimensionless, as is necessary for consistency with Eq. (6).

It is instructive to regard Ω_m satisfying Eq. (6) as the roots corresponding to the intersection of the curve $y_1(\Omega) = G_m =$ constant with the curve

$$y_2(\Omega) = \ln\Omega - \frac{r_m}{(r_m - r_0)}\ln(1 + I\Omega). \tag{9}$$

This latter curve has a single maximum at $\Omega_c = (r_m - r_0)/r_0I$. Also, one finds that the slope $y_2' > 0$ and is monotonic decreasing for $\Omega < \Omega_c$, while $y_2' < 0$ for $\Omega > \Omega_c$, with the limit $y_2' \to 0$ as $\Omega \to \infty$. Furthermore, from Eqs. (2), (6), and (7) one can show that $y_2(\Omega_c) > G_m$ whenever

$$\exp\langle\Omega_m\rangle < 1 + \frac{(\alpha - 1)}{\alpha^{\alpha/(\alpha-1)}}\exp\left[\frac{\alpha\bar{\Omega}_m}{(\alpha - 1)}\right], \tag{10}$$

where $\langle\Omega_m\rangle \equiv 2(r_m - r_0)\langle\sigma_m\rangle/k$, $\bar{\Omega}_m \equiv 2(r_m - r_0)\bar{\sigma}_m/k$, and $\alpha \equiv r_m/r_0$.

Since typically $\alpha \gtrsim 2$, Eq. (10) will hold almost always, and hence there will generally be two roots $\Omega_m^{(1,2)}$ satisfying $y_2[\Omega_m^{(1,2)}] = G_m$ (see the schematic depiction in Fig. 1). This means that both high and low visibility boundary values of extinction can be found for a given return signal, and so the question arises as to how one can discriminate between the two possibilities. Although it is reasonable to expect that the proper choice will

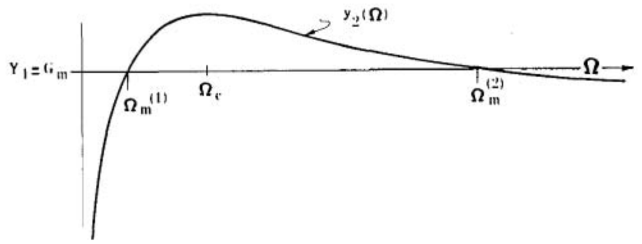

Fig. 1. Plot of the function $y_2(\Omega)$ showing the location of the roots to Eq. (6).

be evident by direct observation under many circumstances, on some occasions (e.g., for highly variable obscurations, under nighttime conditions, or for unattended system operation) the correct choice may not be obvious. Of course, if one knows G_m accurately, presumably the scattering medium has already been characterized as of high or low visibility type, so that further decisions regarding the choice of root may be superfluous, depending largely on how well Eq. (4) reflects the true conditions. If Eq. (4) is a poor approximation, and/or G_m is not known accurately, the ambiguity in the choice of root cannot in principle be removed within the scope of the method described here.

As to the effects of errors δG_m on the estimate for σ_m, from Eq. (6) one finds that for a high-visibility situation the induced fractional error is just $\delta\sigma_m^{(1)}/\sigma_m^{(1)} \simeq \delta G_m$, whereas for low visibilities it is opposite in sign and amplified by the factor $(r_m - r_0)/r_0$: $\delta\sigma_m^{(2)}/\sigma_m^{(2)} \simeq -(r_m - r_0)\delta G_m/r_0$. Because of this behavior a simple half-interval method constitutes an adequate algorithm for locating $\Omega_m^{(1)}$. For $\Omega_m^{(2)}$ a slope extrapolation approach such as the Regula Falsi method[3] is more efficient.

Results of numerical simulations wherein the constant G_m is known indicate that excellent estimates of σ_m may be obtained for optical depths either much greater or much less than unity (i.e., $\Omega_m \gg 1$ or $\Omega_m \ll 1$) so long as σ does not vary systematically by more than a few orders of magnitude over (r_0,r_m) for $\alpha \gtrsim 2$. If $\Omega_m = O(1)$, it may or may not prove difficult to choose the most nearly correct of the two roots, depending on the distribution or $\sigma(r)$. The most difficult cases are for monotonic decreasing σ with r, and this is due primarily to the error introduced by Eq. (4) for such distributions, especially for circumstances limiting α to relatively small values.

Part of this research was conducted while the author was a Senior Research Associate of the National Research Council.

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Mathematical procedure for determining the average irradiance attenuation coefficient of natural waters

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Received 29 September 1982.
0003-6935/83/040515-03$01.00/0.

Although the attenuation of downwelling irradiance in natural waters cannot be confidently taken to be constant until a subsurface asymptotic radiance distribution is established (Preisendorfer,[1] Kirk[2]), the use of an average irradiance attenuation coefficient (k_{av}) to a given irradiance level ($X\%$) should not result in any great loss of generality. For such an assumption, the depth (Z) of this given subsurface irradiance level may be readily obtained from

$$Z = \frac{1}{k_{av}} \cdot \ln \left[\frac{100}{X}\right] . \tag{1}$$

For two distinct values of k_{av} (say, k_{av1} and k_{av2}), the ratio of the corresponding depths (Z_1 and Z_2) to the equivalent subsurface irradiance level is given by

$$\frac{Z_1}{Z_2} = \frac{k_{av2}}{k_{av1}} . \tag{2}$$

Recently we presented[3] the relative depths $Z(\omega,\theta,F)$ of the 1% subsurface irradiance level as a function of the solar zenith angle θ, the fraction F of the above surface incident radiation that is diffuse, and the scattering albedo ω for $\omega = 0.6$, 0.75, and 0.9 (see Tables III, IV, and V of Jerome *et al.*[3]). From the work of Whitney[4] (which assumed no scattering phenomena occurring within the water column) a similar table may be constructed for $\omega = 0$. Table I lists the resulting relative depths $Z(0,\theta,F)$ of the 1% subsurface irradiance level as a function of θ and F for $\omega = 0$. For convenience, Tables II, III, and IV (taken from Tables III, IV, and V of our previous work[3]) are reproduced to illustrate the comparable values for $\omega = 0.6$, $\omega = 0.75$, and $\omega = 0.9$, respectively.

For a particular water mass ω, the relationship between k_{av1} and k_{av2} (defined by two sets of θ and F values) is given by

$$\frac{k_{av1}}{k_{av2}} = \frac{k_{av}(\omega,\theta_1,F_1)}{k_{av}(\omega,\theta_2,F_2)} = \frac{Z(\omega,\theta_2,F_2)}{Z(\omega,\theta_1,F_1)} \tag{3}$$

and may be applied to the values listed in Table I, Table II, Table III, or Table IV. Interpolation for values of θ and F not listed in the tables should not result in inaccuracies exceeding ~5%, the largest inaccuracies being associated with midrange solar zenith angles, totally direct incident radiation, and low scattering albedo.

Thus, if an *in situ* determination of k_{av} to the 1% subsurface irradiance level is performed for a particular water mass ω and a particular set of conditions θ_1 and F_1, the absolute value of k_{av} for that water mass can be estimated for any other set of θ and F conditions from Eq. (3) and the appropriate values of $Z(\omega,\theta,F)$.

However, since the values of $Z(\omega,\theta,F)$ listed in Tables I–IV have been normalized to $Z(\omega,0,0)$, the absolute values of $Z(\omega,0,0)$ must be known before intercomparisons between distinct water masses (i.e., water masses of different ω values) may be considered. Alternatively, the absolute value of $k_{av}(\omega,0,0)$ for each water mass must be known.

Kirk[2] has utilized a Monte Carlo simulation of the propagation of radiation in natural waters to estimate the influence