Analysis of atmospheric lidar observations: some comments

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There have been many discussions of solutions to the lidar equation for elastic scattering (e.g., Fernald *et al.*,[1] Klett,[2] Davis,[3] and Collis and Russell[4]). Most of these are simply variations on Hitschfeld and Bordan's[5] solution for meteorological radars. Klett[2] recently restated this solution in a very convenient form for the analysis of lidar observations collected in very turbid atmospheres. His paper has prompted a restatement of the more general solution of Fernald *et al.*[1] which is also applicable to mildly turbid atmospheres where both aerosol and molecular scatterers must be considered in the analysis. This has led to a simple numerical scheme for the computer analysis of lidar measurements.

The lidar equation for two distinct classes of scatters (Fernald *et al.*[1]) is

$$P(Z) = ECZ^{-2}[\beta_1(Z) + \beta_2(Z)]T_1^2(Z)T_2^2(Z), \quad (1)$$

where $P(Z)$ = the return signal that is proportional to the received power from a scattering volume at slant range Z,

E = an output energy monitor pulse which is proportional to the transmitted energy,

C = the calibration constant of the instrument which includes losses in the transmitting and receiving optics and the effective receiver aperture,

$\beta_1(Z)$ and $\beta_2(Z)$ = respectively, the backscattering cross sections of the aerosols and molecules at slant range Z,

$T_1(Z) = \exp[-\int_0^Z \sigma_1(z)dz]$ = the aerosol transmittance,

$T_2(Z) = \exp[-\int_0^Z \sigma_2(z)dz]$ = the molecular atmosphere transmittance, and

where $\sigma_1(Z)$ and $\sigma_2(Z)$ = respectively, the extinction cross sections of the aerosols and molecules at range Z.

The molecular atmosphere scattering properties, $\beta_2(Z)$ and $\sigma_2(Z)$, can be determined from the best available meteorological data or approximated from appropriate standard atmospheres; so that only the aerosol scattering properties, $\beta_1(Z)$ and $\sigma_1(Z)$, remain to be determined. One further simplifying assumption is that the extinction-to-backscattering ratio for aerosols, $S_1 = \sigma_1(Z)/\beta_1(Z)$, remains constant with range. It essentially states that the size distribution and composition of the aerosol scatterers are not changing with range from the lidar, and that variations in backscattering from aerosols are due to changes in their number density. This is not exceedingly restrictive. In the numerical analysis of lidar data, the atmosphere can be divided into layers, with S_1 allowed to vary among the layers. Collis and Russell,[4] Pinnick *et al.*,[6] and Russell *et al.*[7] can be referenced for values for this ratio. The corresponding ratio for the molecular scatterers is the constant $S_2 = \sigma_2(Z)/\beta_2(Z) = 8\pi/3$.

The solution to Eq. (1) for the aerosol backscattering cross sections (Fernald *et al.*[1]) then becomes

$$\beta_1(Z) = \frac{P(Z)Z^2 \exp[-2(S_1 - S_2)\int_0^Z \beta_2(z)dz]}{CE - 2S_1\int_0^Z P(z)z^2 \exp[-2(S_1 - S_2)\int_0^z \beta_2(z')dz']dz} - \beta_2(Z). \quad (2)$$

If *a priori* information can be used to specify the value of the aerosol and molecular scattering cross sections at a specific range Z_c, the lidar can be calibrated by solving Eq. (2) for CE in terms of these scattering properties and

$$\beta_1(Z) + \beta_2(Z) = \frac{X(Z)\exp[-2(S_1 - S_2)\int_{Z_c}^Z \beta_2(z)dz]}{\frac{X(Z_c)}{\beta_1(Z_c) + \beta_2(Z_c)} - 2S_1\int_{Z_c}^Z X(z)\exp[-2(S_1 - S_2)\int_{Z_c}^z \beta_2(z')d']dz}, \quad (3)$$

where $X(Z)$ is the range normalized signal $P(Z)Z$.[2] The total backscattering cross section at range Z is now expressed as a function of the scattering properties at the calibration range and those of the intervening atmosphere between the ranges Z_c and Z.

Equation (3) leads to a simple numerical integration scheme. If

$$A(I,I+1) = (S_1 - S_2)[\beta_2(I) + \beta_2(I+1)]\Delta Z \quad (4)$$

is used to replace the exponential terms that incorporate the effects of aerosol extinction between adjacent data points range ΔZ apart, the total backscattering cross section at range $Z(I+1)$, one data step beyond the calibration range $Z(I)$, becomes

$$\beta_1(I+1) + \beta_2(I+1) = \frac{X(I+1)\exp[-A(I,I+1)]}{\frac{X(I)}{\beta_1(I) + \beta_2(I)} - S_1|X(I) + X(I+1)\exp[-A(I,I+1)]|\Delta Z}. \quad (5)$$

Similarly, the total backscattering cross section at $Z(I-1)$, one step before the calibration range $Z(I)$, becomes

$$\beta_1(I-1) + \beta_2(I-1) = \frac{X(I-1)\exp[+A(I-1,I)]}{\frac{X(I)}{\beta_1(I) + \beta_2(I)} + S_1|X(I) + X(I-1)\exp[+A(I-1,I)]|\Delta Z}. \quad (6)$$

Solutions in terms of aerosol extinction are correspondingly

$$\sigma_1(I+1) + \frac{S_1}{S_2}\sigma_2(I+1) = \frac{X(I+1)\exp[-A(I,I+1)]}{\frac{X(I)}{\sigma_1(I) + S_1/S_2\sigma_2(I)} - |X(I) + X(I+1)\exp[-A(I,I+1)]|\Delta Z}, \quad (7)$$

$$\sigma_1(I-1) + \frac{S_1}{S_2}\sigma_2(I-1) = \frac{X(I-1)\exp[+A(I-1,I)]}{\frac{X(I)}{\sigma_1(I) + S_1/S_2\sigma_2(I)} + |X(I) + X(I-1)\exp[+A(I-1,I)]|\Delta Z}. \quad (8)$$

The lidar data can, therefore, be analyzed in successive steps that can move either out or in from the assigned calibration range.

Some general comments can now be made concerning the application of Eqs. (5)–(8) to different atmospheric conditions. They are dependent on the laser wavelength, the extent to which multiple scattering can be ignored, and the data sampling interval ΔZ of the specific lidar system being used. The conclusions concerning highly turbid atmosphere are basically a reiteration of those of Klett.[2]

For highly turbid atmosphers ($\sigma_1 \gg \sigma_2$), the molecular scatterers can be ignored and Eqs. (6) and (8) reduce to Klett's.[2] In these atmospheric conditions the two terms in the denominators will be of comparable magnitude so that outward stepwise integration, Eqs. (5) and (7), can become very unstable. On the other hand, inward stepwise integration is very stable and rapidly loses its dependence on the initial guess of the scattering cross sections attributed to the calibration range. In this sense, uncalibrated lidars can yield the extinction properties of highly turbid atmospheres. Equations (6) and (8) for highly turbid atmospheres become

$$\beta_1(I-1) = \frac{X(I-1)}{\frac{X(I)}{\beta_1(I)} + S_1[X(I)+X(I-1)]\Delta Z}, \tag{9}$$

$$\sigma_1(I-1) = \frac{X(I-1)}{\frac{X(I)}{\sigma_1(I)} + [X(I)+X(I-1)]\Delta Z}. \tag{10}$$

When the aerosol and molecular scattering cross sections are of a comparable magnitude (during light to moderate air pollution events or in stratospheric studies), the second terms in the denominators of Eqs. (5)–(8) will be considerably smaller than the first terms. Numerically stable solutions are, therefore, possible when stepping in either direction from the calibration level. In these atmospheric conditions, the analyses will be dependent on the aerosol and molecular backscattering cross sections assigned to the calibration level. Net aerosol extinction will be small. It will be tied to the values selected for S_1, the aerosol extinction-to-backscattering ratio which can vary over a relatively wide range without greatly affecting the backscattering cross sections computed for Eqs. (5) and (6).

The analyses developed above lend themselves readily to qualitative statements collected in highly turbid, moderately turbid, and relatively clean atmosphers. The precise definition of these atmospheres will vary among lidar systems, primarily with the laser wavelength and data sampling interval.

In highly turbid atmospheres, aerosols dominate the scattering process to the extent that molecular scattering can be ignored. From Eq. (10) it can be demonstrated that an uncalibrated lidar can readily yield aerosol extinction profiles. On the other hand, backscattering profiles, Eq. (9), are directly dependent on an accurate knowledge of the extinction-to-backscattering ratio S_1.

In relatively clean atmospheres, the basic result of the analysis is the aerosol backscattering cross section, and the aerosol extinction now becomes dependent on an accurate knowledge of the extinction-to-backscattering ratio.

For moderately turbid atmospheres, lying in some ill-defined region between the two cases discussed above, the analyses will be sensitive to both the extinction and backscattering properties of the aerosols. The lidar system must be accurately calibrated, and the extinction-to-backscattering ratio must be reasonably well established. Equations (5)–(8) lend themselves to a very compact sequence of FORTRAN statements for the computer analysis of digitized lidar observations.

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Atmospheric temperature measurement using a pure rotational Raman lidar: comment

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The authors of this paper[1] are to be congratulated. The success of the effort along with the punctilious attention to detail shown therein has greatly enhanced the development of methods for acquisition of accurate atmospheric temperature profiles.

The purpose of this Letter is to comment on the severity of the elastic cross talk entering the Raman channels and to suggest why such cross talk is a matter of somewhat less concern than the authors suppose.

As noted in Ref. 1 the presence of an enhanced aerosol overburden increases both the elastic backscatter and transmission losses. Due to the finite rejection of radiation at the elastic wavelength (Rayleigh and Mie backscatter) in the Raman channels some of the so-called online radiation penetrates and becomes part of the signal acquired in the Raman channel. The greater the aerosol overburden the greater the penetration into the Raman channel and so the greater the amount of unwanted signal which can act to reduce the accuracy of the measurement.

It is noted in Ref. 1 that because of the ratio of the Raman to Rayleigh cross section along with nominal overburdens ($\simeq$10.0-km visibility), and because of the need to keep the errors on the temperature measurement below the 1.0% level, transmission of the elastic line in the Raman channel must be of the order of 10^{-6}. For the small wavelength intervals which separate the pertinent Raman lines from the elastic line rejection by factors of 10^6 are quite difficult to achieve by ordinary narrowband interference filters. When using a ruby laser (6943 Å) as the existing source useful Raman lines suggest that the Raman channels be set in the vicinity of 6910 and 5990 Å. Interference filters were used in a series of experiments[2] which led to temperature uncertainties of $\simeq$1.0 K and an absolute temperature difference of mean value integrated all along the profile to 2.3 km of $\simeq$2.5 K between lidar and radiosonde methods of profile acquisition. Here, however, the filter rejection was only $\sim 10^5$.

The seeming inconsistency of the 1% error requirement of 10^6 rejection, on the one hand, and the <1% error obtained using a filter with rejection of 10^5, on the other, is resolved in the following way. The temperature measurement itself is given by the ratio of the intensities in the two Raman channels. For the moment assume for the sake of clarity of ex-